Modeling Engineering Applications Using Simulink

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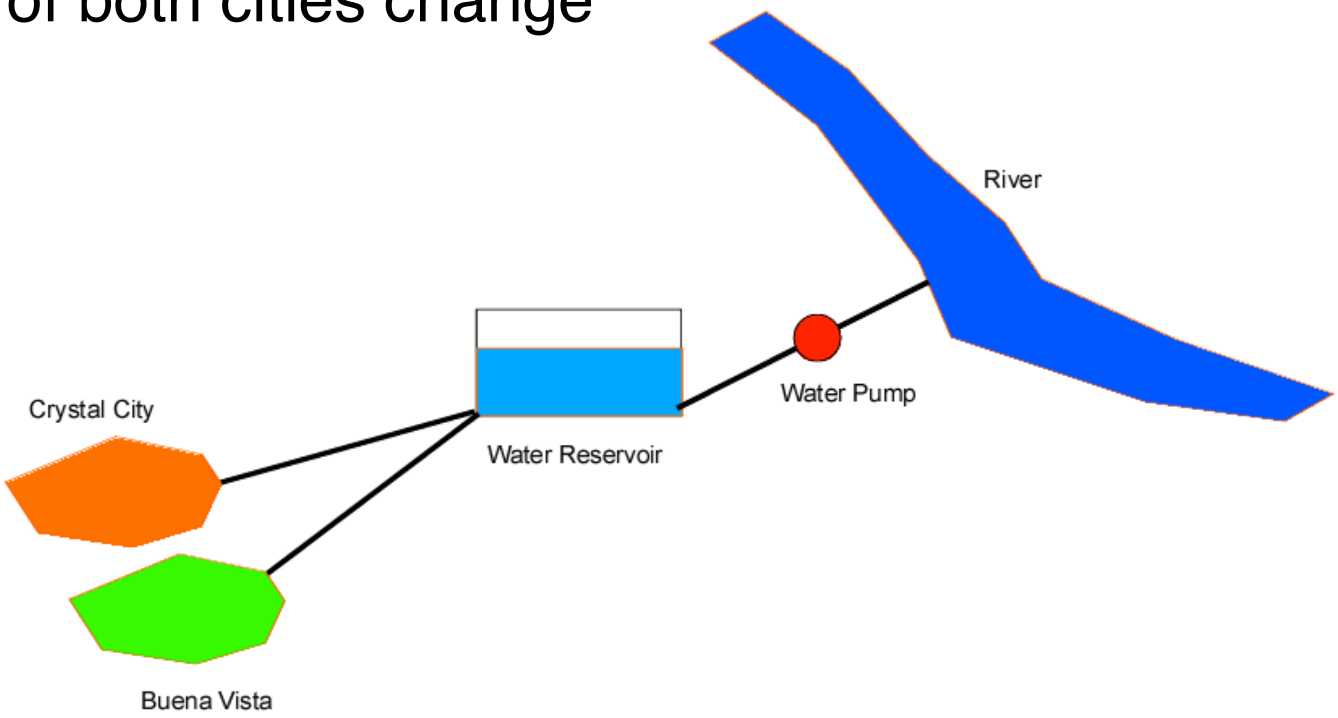
CEE 3804

Computer Applications in CEE

Modeling A Water Distribution System Using Simulink

Water Distribution System

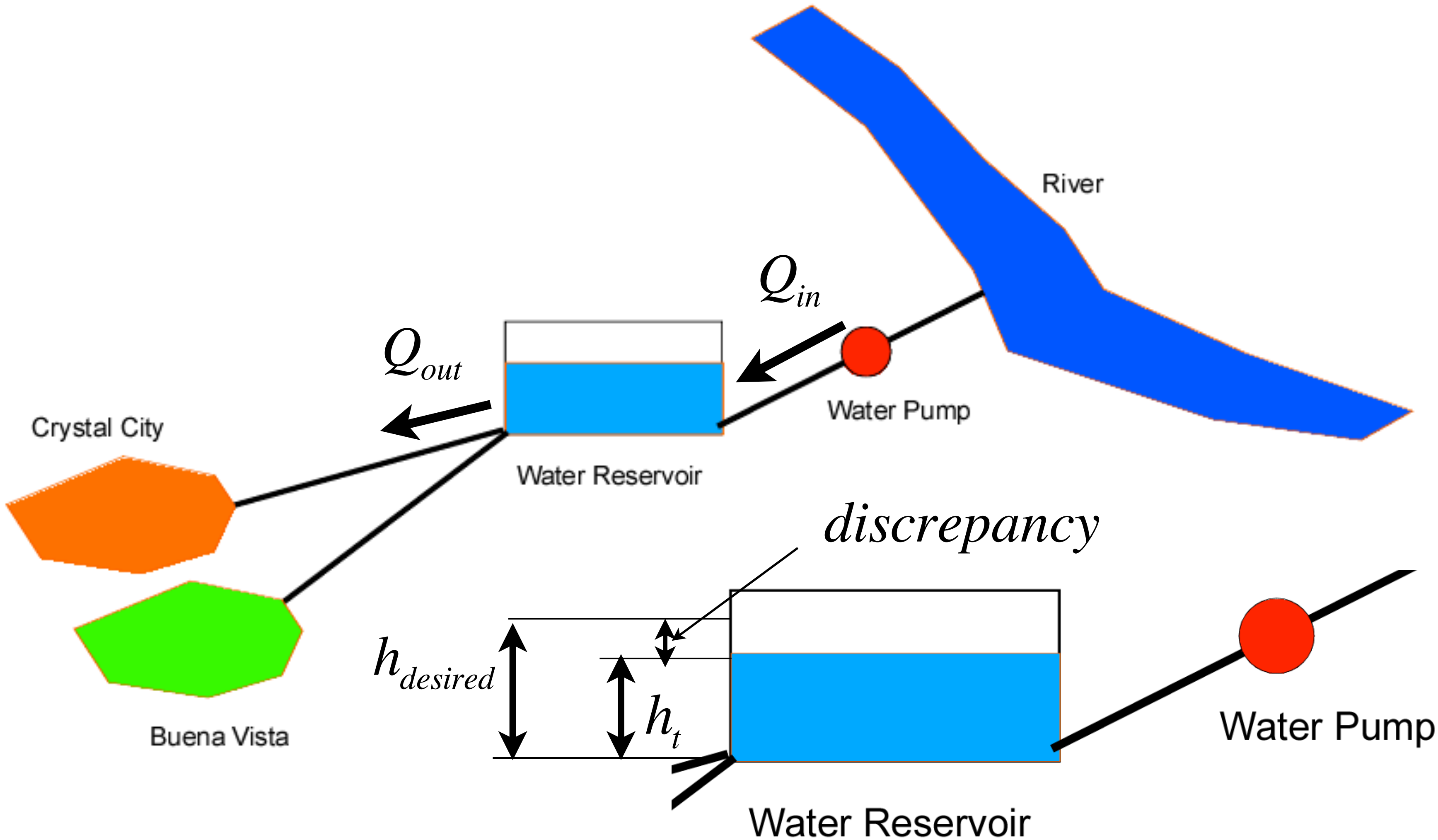
- Water is stored in a tank with height h and area a
- The water for the tank is pumped (using mechanical means) to the tank
- Water is distributed to two cities from the tank as the demands of both cities change



Water Control System

- The water height (called “head” in hydraulics) in the reservoir is controlled by pumping water form the river to the reservoir
- The idea is to maintain a desired head (height) in the reservoir providing enough water pressure for distribution to the cities (Buena Vista and Crystal City)
- A simple “algorithm” is to maintain a constant head in the reservoir to some desired height ($h_{desired}$)
- If the water level in the reservoir falls below the desired head, then water is pumped according to the difference between the actual height (h_t) and the desired height

Details of Water Control System



Basic Equations

$$\frac{dVol_t}{dt} = Q_{in} - Q_{out}$$

Rate of change of volume of reservoir

$$Q_{out} = Q_{CrystalCity} + Q_{BuenaVista}$$

Demand volumes

$$Q_{in} = disc * k_{pump}$$

Volume into reservoir

$$disc = h_{desired} - h_t$$

Discrepancy of head

$$h_t = Vol_t / area$$

Head of reservoir

The values of area, desired head and demand volumes are known

Steady-State Analysis

- Look at the system as time goes to infinity
- Rates of change of the system are in equilibrium

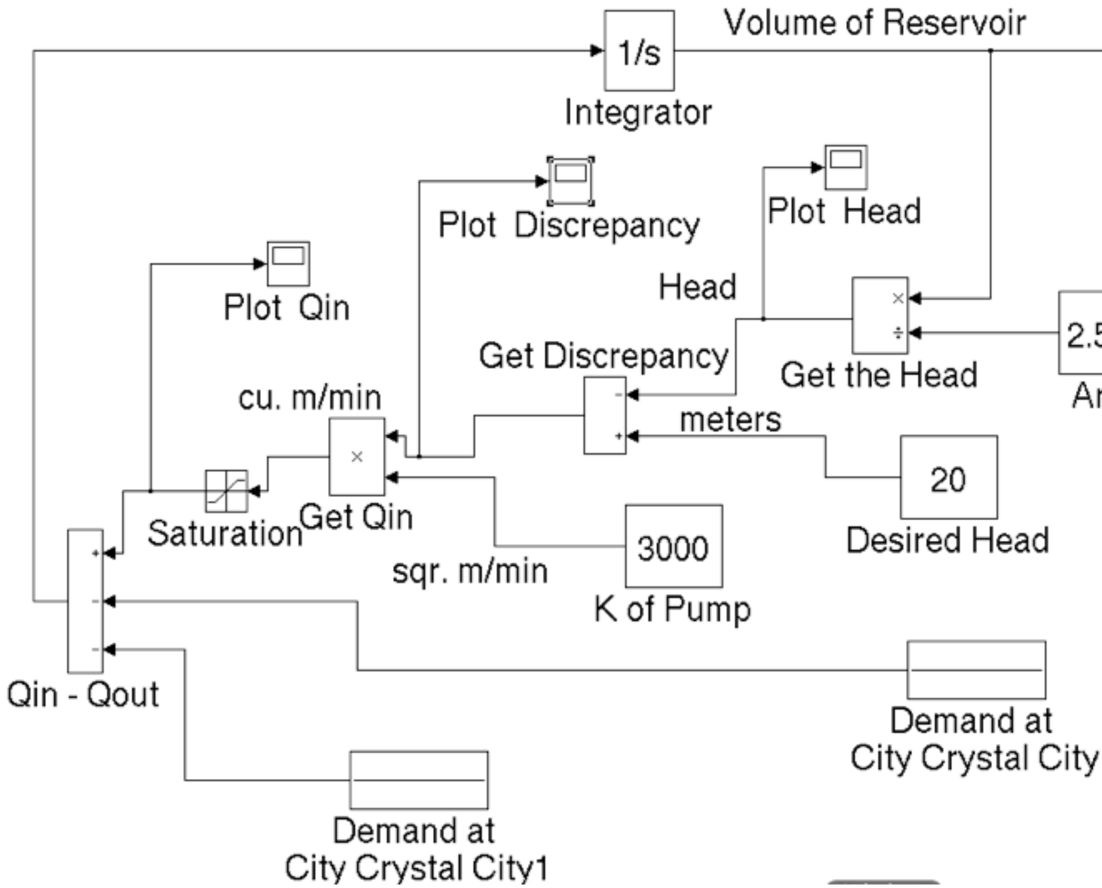
$$\frac{dVol_t}{dt} = Q_{in} - Q_{out} = 0$$

$$Q_{out} = Q_{CrystalCity} + Q_{BuenaVista} = disc * k_{pump}$$

$$Q_{CrystalCity} + Q_{BuenaVista} = (h_{desired} - Vol_e / area) * k_{pump}$$

Solve for Volume at equilibrium or head to obtain desired steady-state conditions

Baseline Simulink Model



Equations of the water control

Qin = disc * Kpump

Qout = f(time)

dVol/dt = Qin - Qout

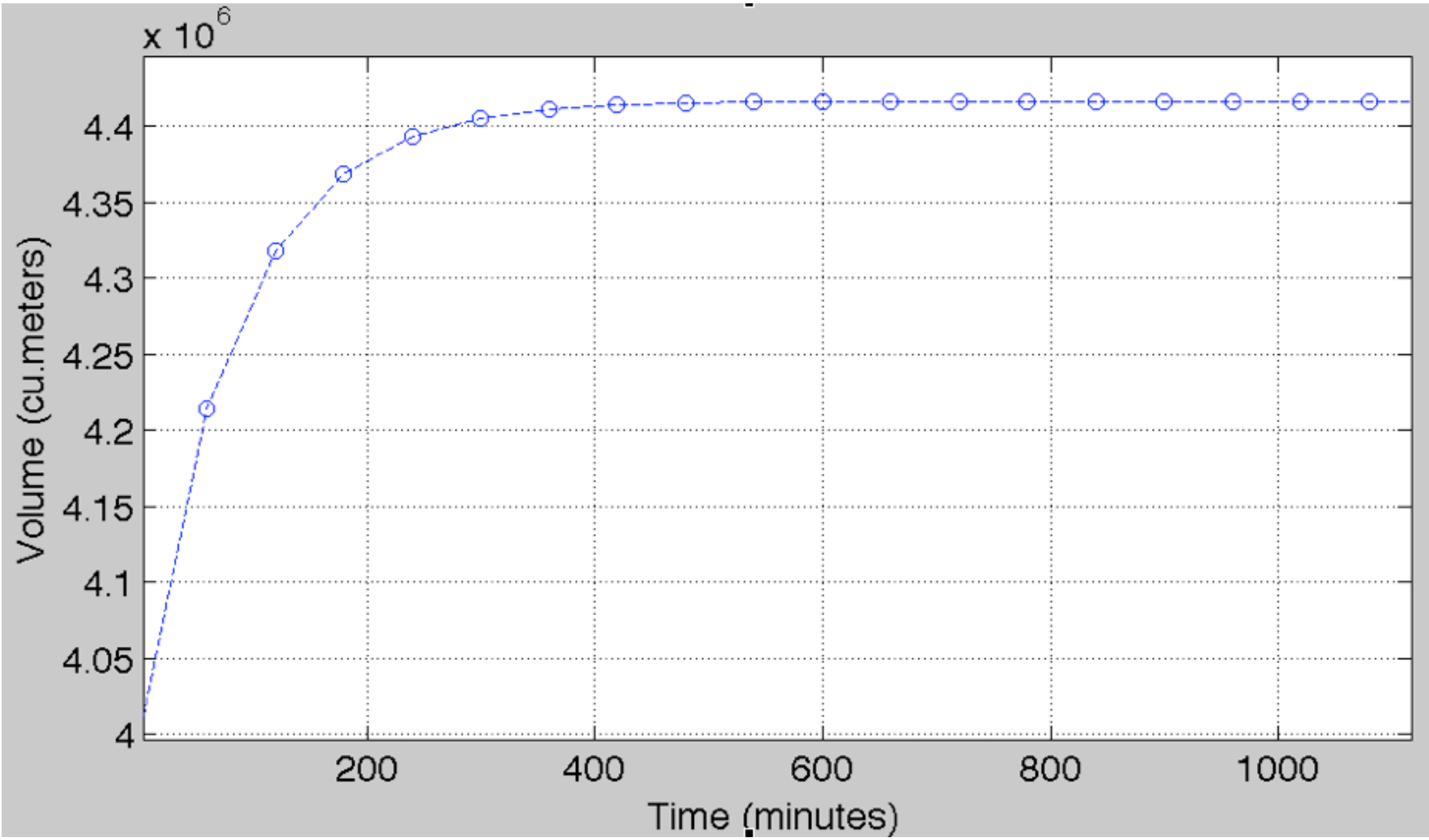
disc = Desired Head - Head

Head = Volume / Area

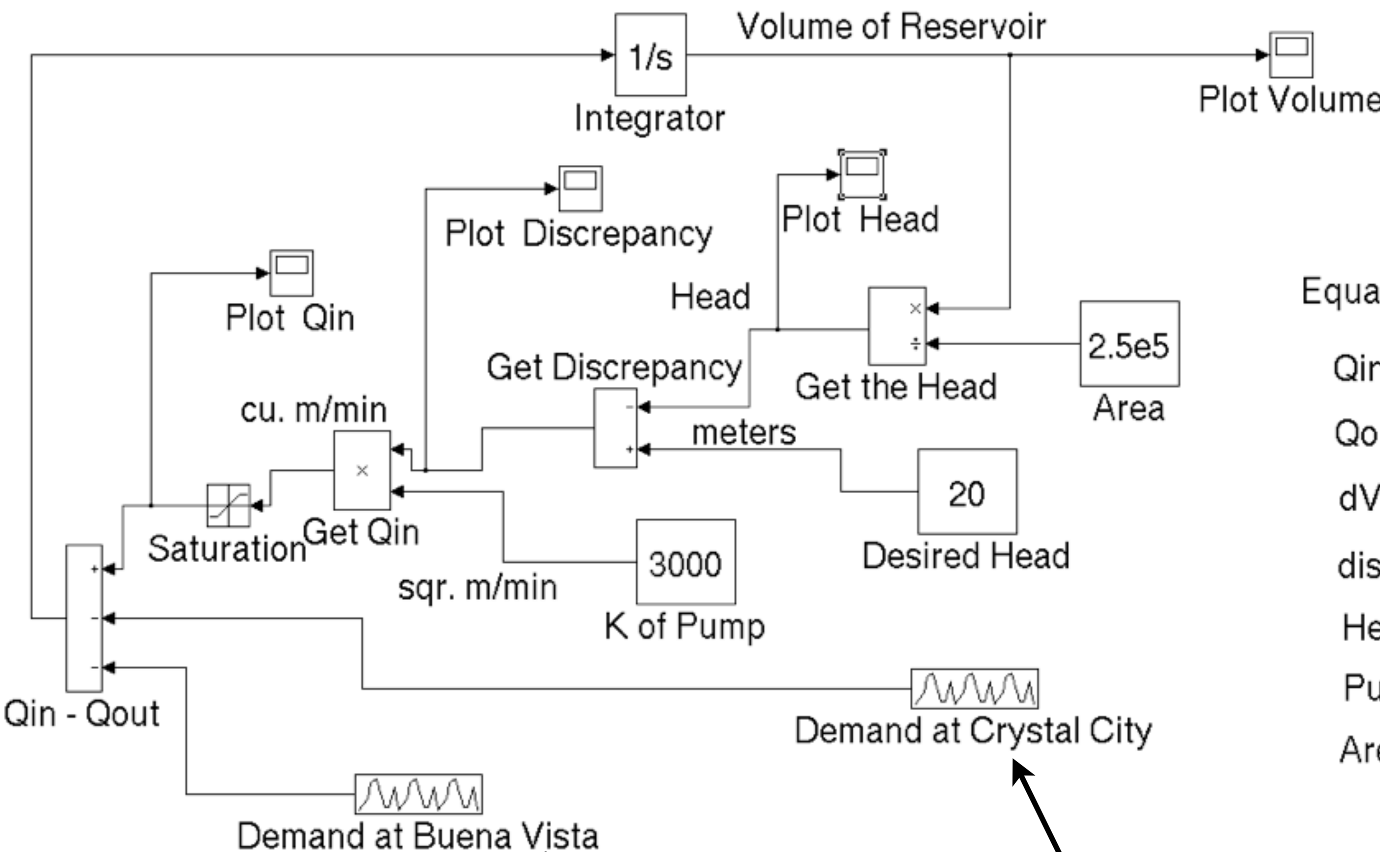
Pump constant = 3000

Area = 2.5e5 m-m

Dynamic Behavior of Baseline Model (Volume vs. Time)



Modified Simulink Model



Equations of the water control

Qin = disc * Kpump

Qout = f(time)

dVol/dt = Qin - Qout

disc = Desired Head - Head

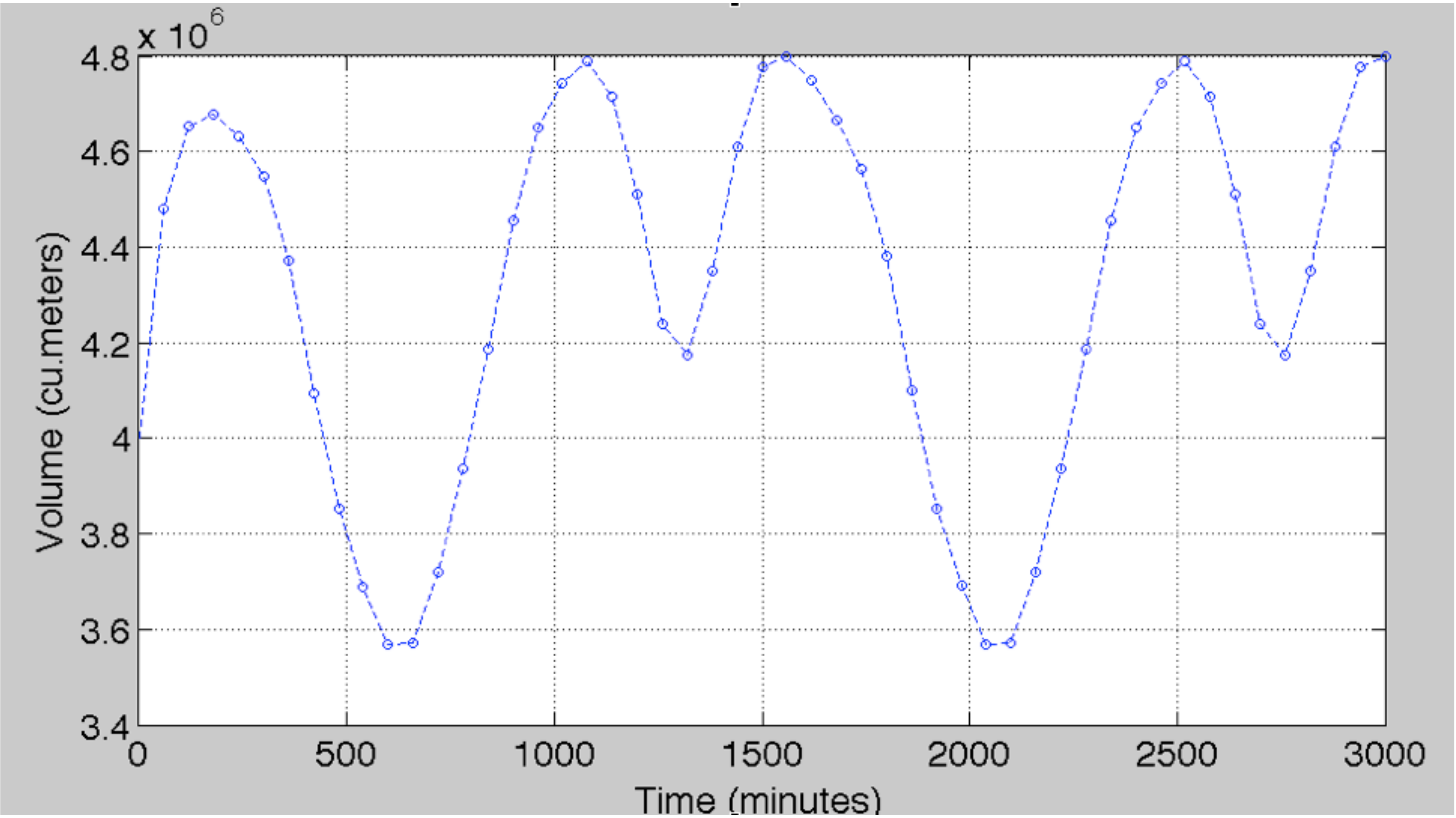
Head = Volume / Area

Pump constant = 3000

Area = 2.5e5 m-m

Time dependent demands at cities

Dynamic Behavior of Updated Model (Volume vs. Time)



Modeling a Runway Arrestor System Using Simulink

Description

- A runway arrestor is a “soft concrete” (also called cellular concrete) bed installed at the end of runway to stop an aircraft involved in a runway overrun
- Overruns occur because in rare occasions aircraft abort the takeoff maneuver
- Overruns can also occur because aircraft land “long” on the runway and then do not have enough distance to stop before the end of the runway
- Check out an accident at Burbank, California

Example Installation of Runway Arrestor Bed

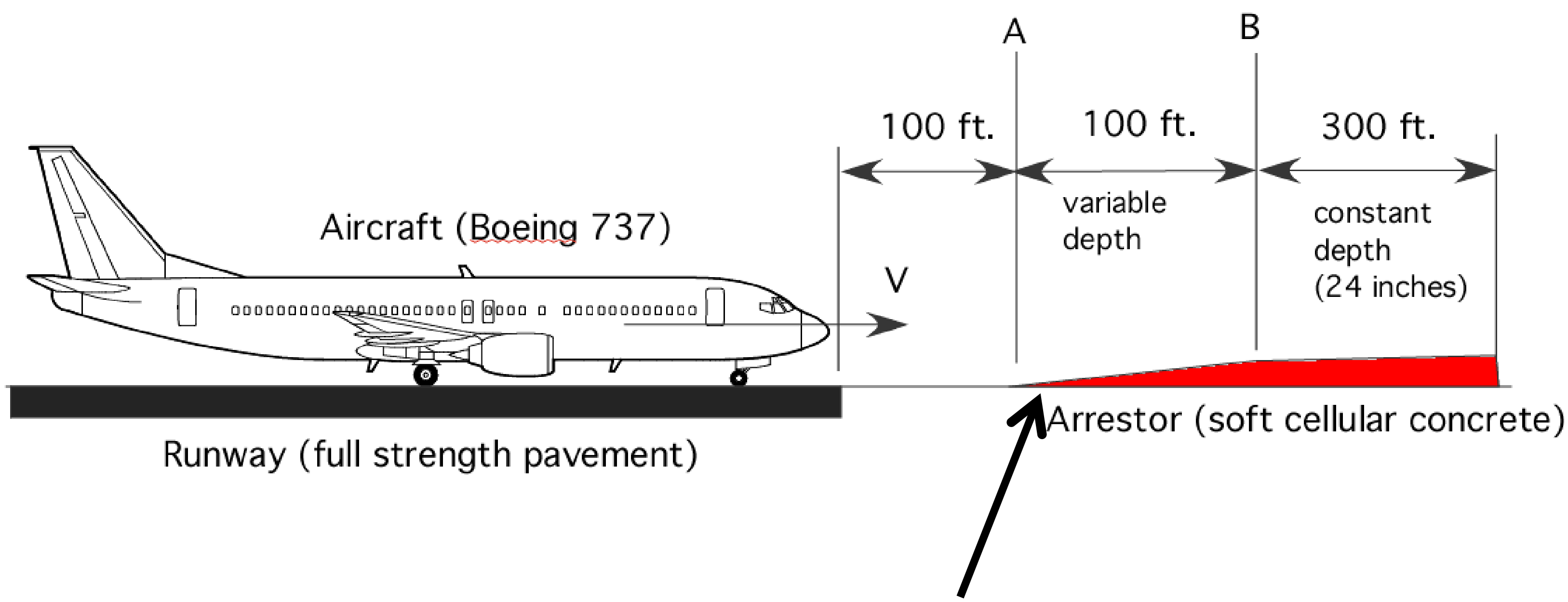
Virginia Tech

Sample Overruns in the U.S (check them out)

- May 1999: A Saab 340 commuter aircraft overran the runway at JFK
- May 2003: Gemini Cargo MD-11 overran the runway at JFK
- January 2005: A Boeing 747 overran the runway at JFK (http://www.airliners.net/open.file?id=0764263&size=L&width=1024&height=774&sok=&photo_nr=)
- December 2005: Boeing 737-700 Southwest Airlines at Midway airport
- July 2006: Falcon 900 airplane overran the runway at the Greenville Downtown Airport in South Carolina

Boeing 737 Runway Overrun
(source: National Transportation Safety Board)

Typical Installation
(for Medium Transport Aircraft - Boeing 737)

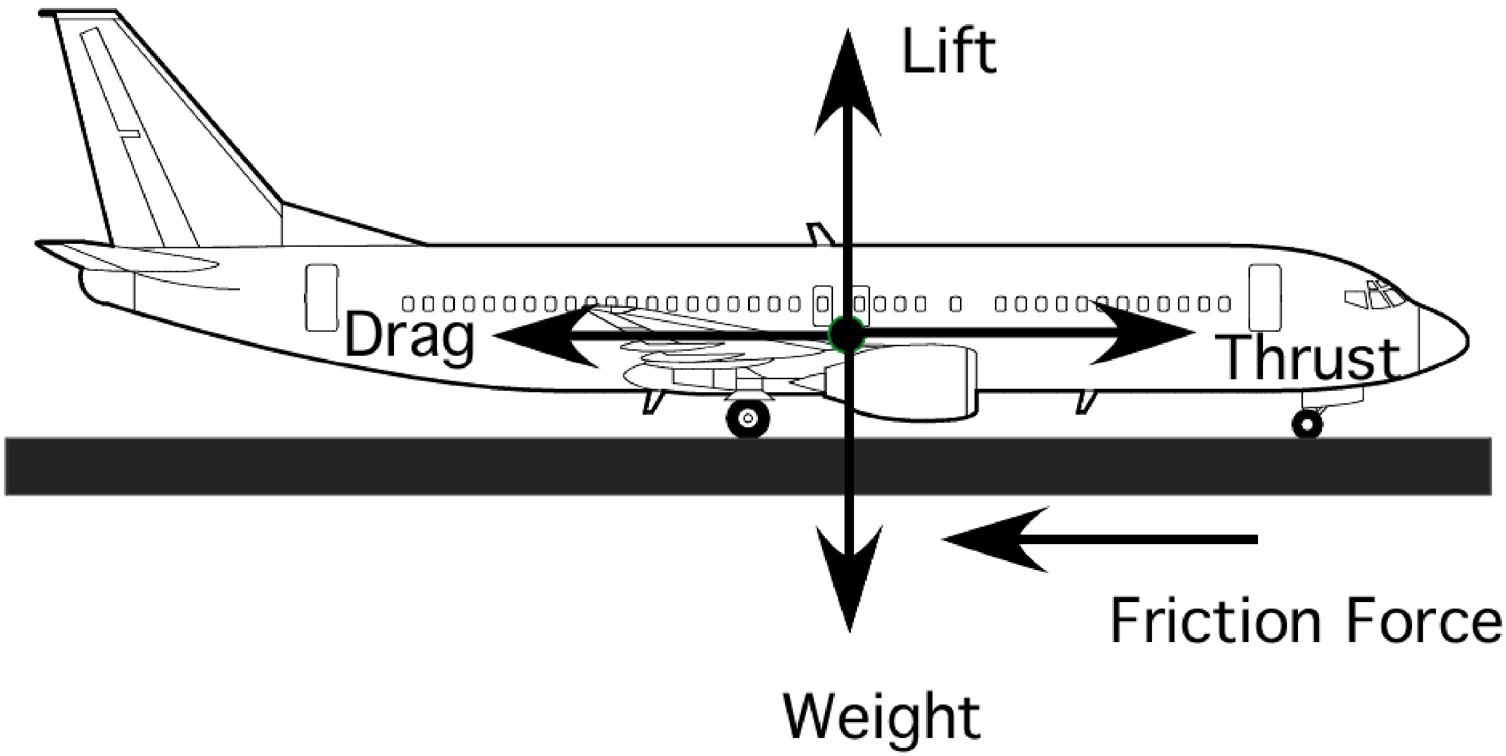


As the aircraft enters the arrestor bed the landing gear is subjected to larger friction forces

(as the tires penetrate the cellular concrete)

Ideas for Modeling

- Consider all possible forces acting on the aircraft



Functional Form of Forces Acting on the Aircraft

Virginia Tech 1872

The functional form of these forces has been known to be of the form,

$$L = \frac{1}{2}\rho V^2 S C_L \quad \textbf{(EQ 1)}$$

$$D = \frac{1}{2}\rho V^2 S C_D \quad \textbf{(EQ 2)}$$

$$T = f(V, \rho) \quad \textbf{(EQ 3)}$$

$$F_f = (mg\cos\phi - L)\mu \quad \textbf{(EQ 4)}$$

V is the vehicle speed (m/s), ρ is the air density (kg/m^3), S is the aircraft gross wing area, C_L is the lift coefficient (non dimensional), C_D is the drag coefficient (non dimensional), μ is the coefficient of friction (non dimensional), and ϕ is the angle formed between the rolling terrain ahead of the aircraft and the horizontal plane.

Using Newton's second law and summing forces in the horizontal direction of motion (x),

$$ma_x = T(V, \rho) - D - (mg\cos\phi - L)\mu - mg\sin\phi \quad \textbf{(EQ 5)}$$

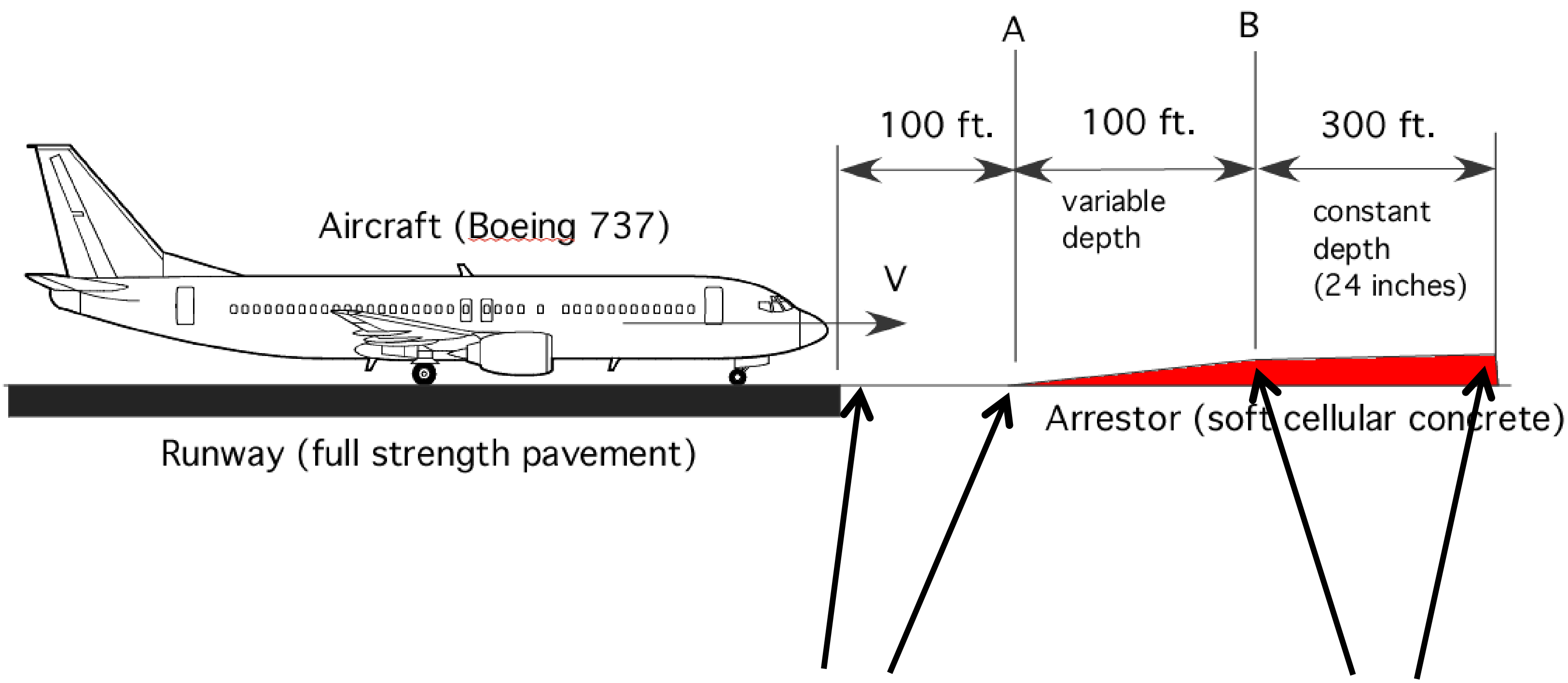
Lets Simplify the Problem

- Lift is very small at a typical runway exit speed (70 miles per hour)
- Drag is small as well
- Assume no thrust reverser is used (near zero thrust). This can be an important scenario because thrust reverses some times are inoperative
- Weight and friction forces are the only remaining forces in the problem
- Assume a flat runway for the first iteration of the model

First Computer Model

- Since the arrestor has variable geometry it is expected that it will induce a variable value of coefficient of friction (μ)
- $m * a_x = - m * g * \mu$
- $a_x = - g * \mu$
- Empirical studies provide some insight into the values of (μ)

Values of Coefficient of Friction (Empirical Study)



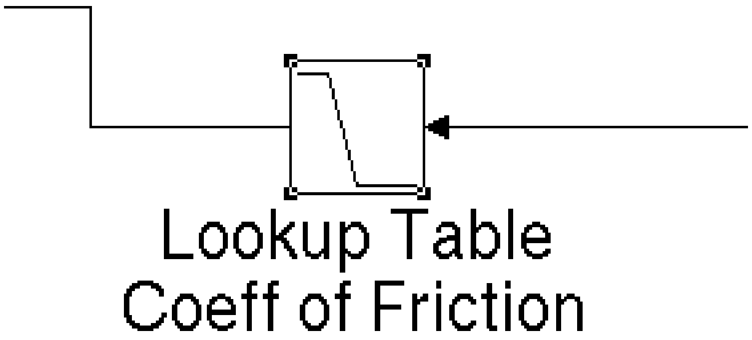
$\mu = 0.03$ $\mu = 0.56$

Parameters of the Model

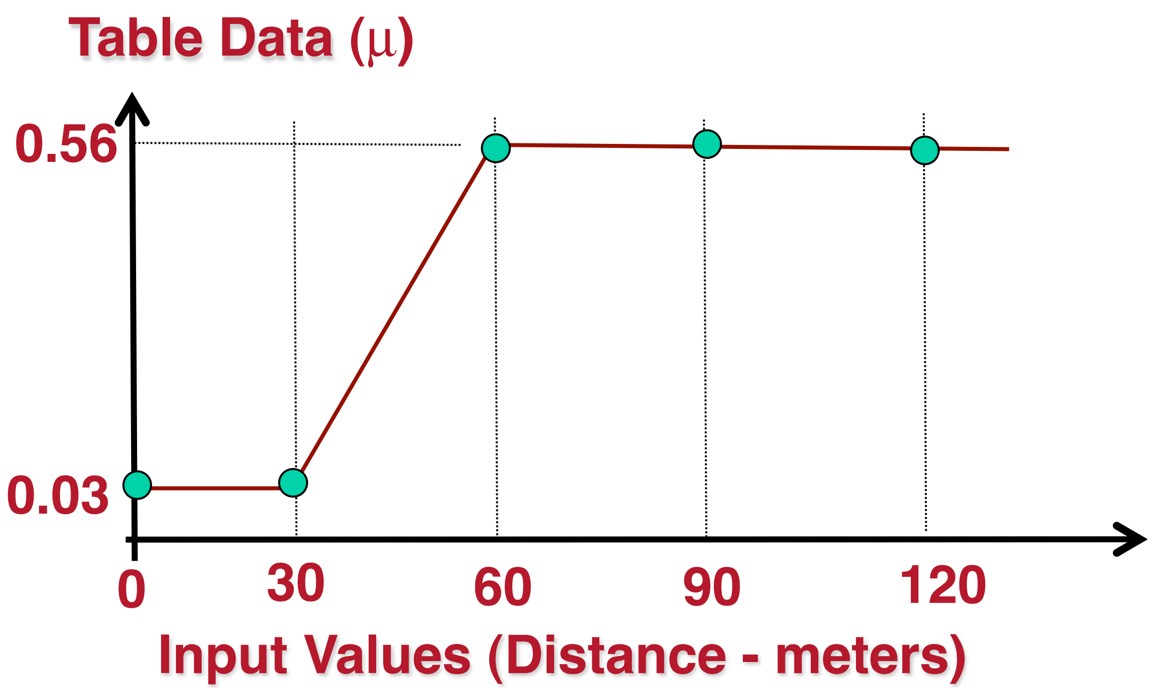
- The Boeing 737-700 has a mass of 76,000 kg (167,000 lb)
- The acceleration of gravity is 9.81 m/s-s
- The arrestor bed (called an EMAS) induces a variable coefficient of friction **(μ)** as the aircraft travels over the arrestor bed
- Assume a linear variation in **(μ)** between the start of the arrestor ramp and the constant section (see the diagram in previous slide)
- Equation to be solved
- $\mathbf{a_x = dV/dt = - g} * \mu(t)$
- where (μ) varies with position of the aircraft
- Create a Simulink model to plot the speed and distance profile of the aircraft

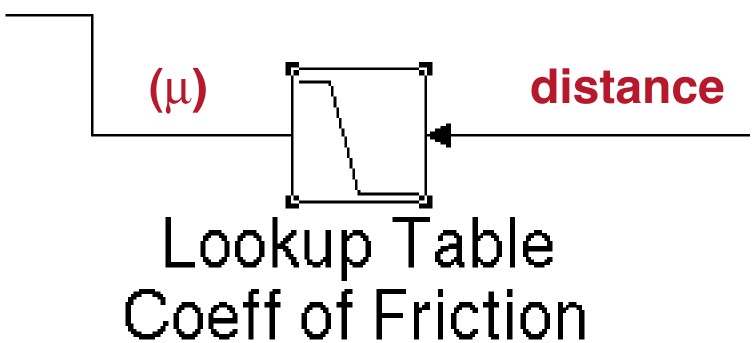
Table Look-up Functions

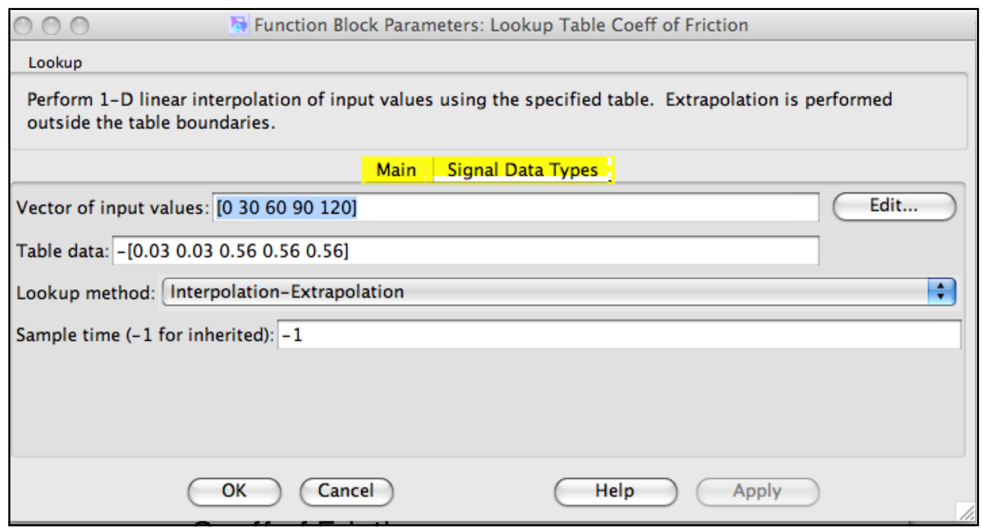
- Provide a flexible mechanism to model a causal relationship between two or more variables
- For example: the coefficient of friction (μ) is a function of distance (since the arrestor bed increases in thickness)
- In Simulink Table look-up function blocks are located in the lookup table library



Sample Look-up Table





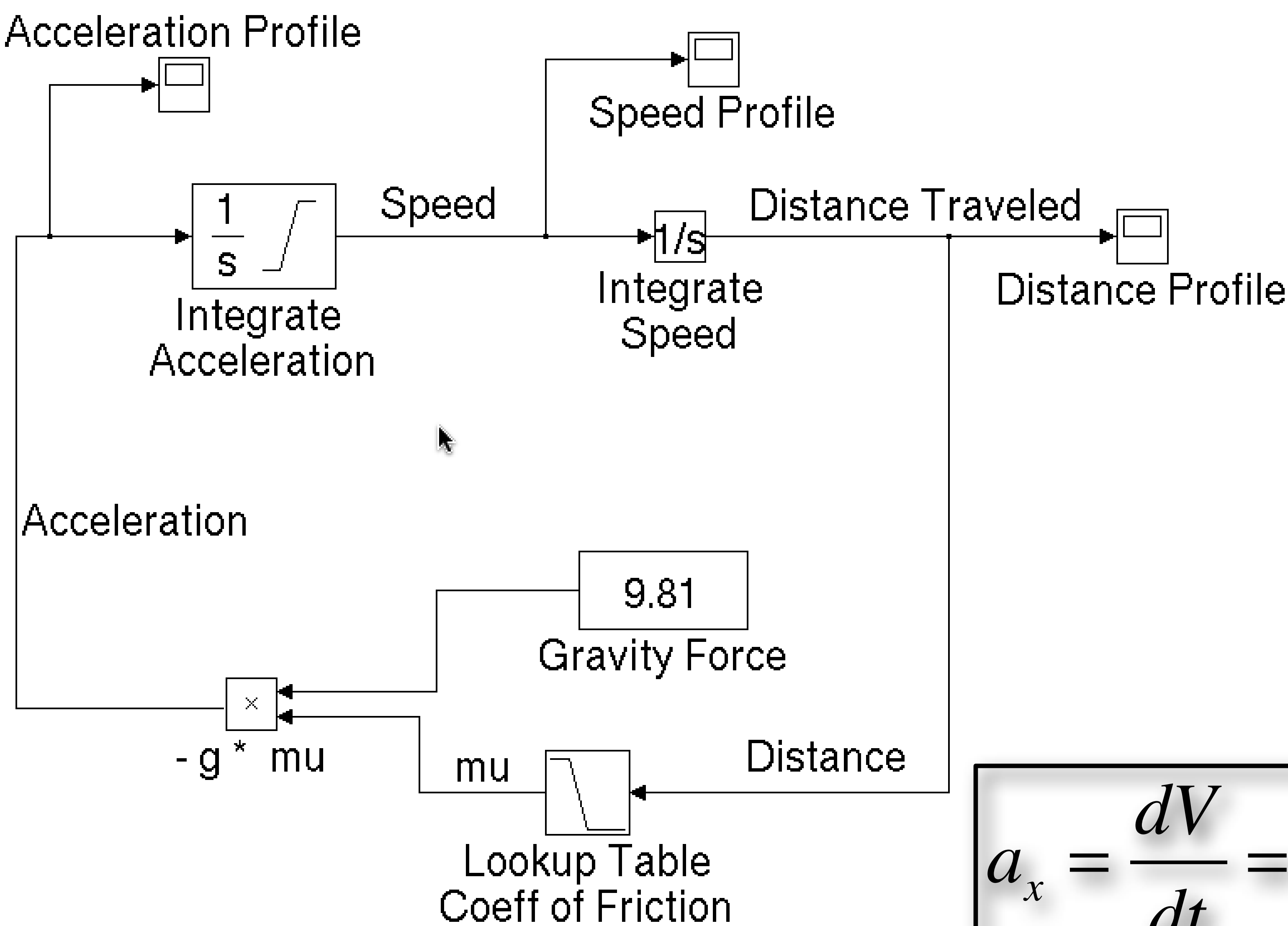


Lets Do It in Class

- Discuss possible approaches to model the variable coefficient of friction
- Improve the model to add thrust forces

- Find:
- The distance to stop an aircraft traveling at 30 m/s when it leaves the runway
- Repeat for 25, 20 and 15 m/s

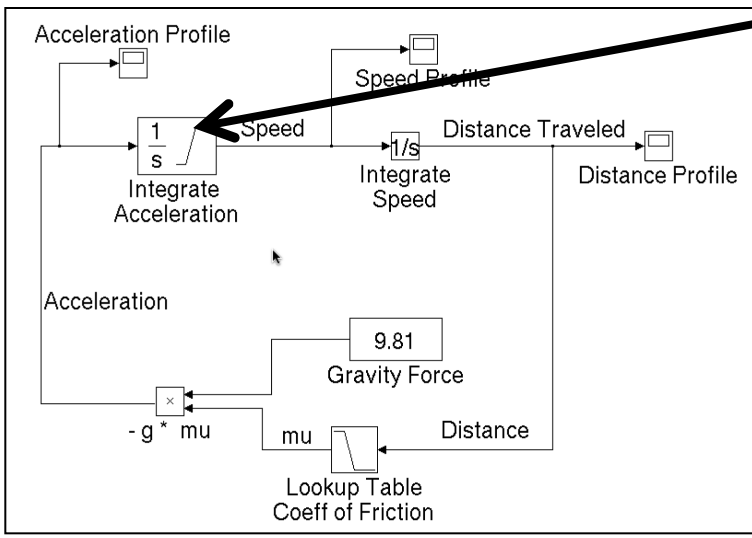
Simulink Model

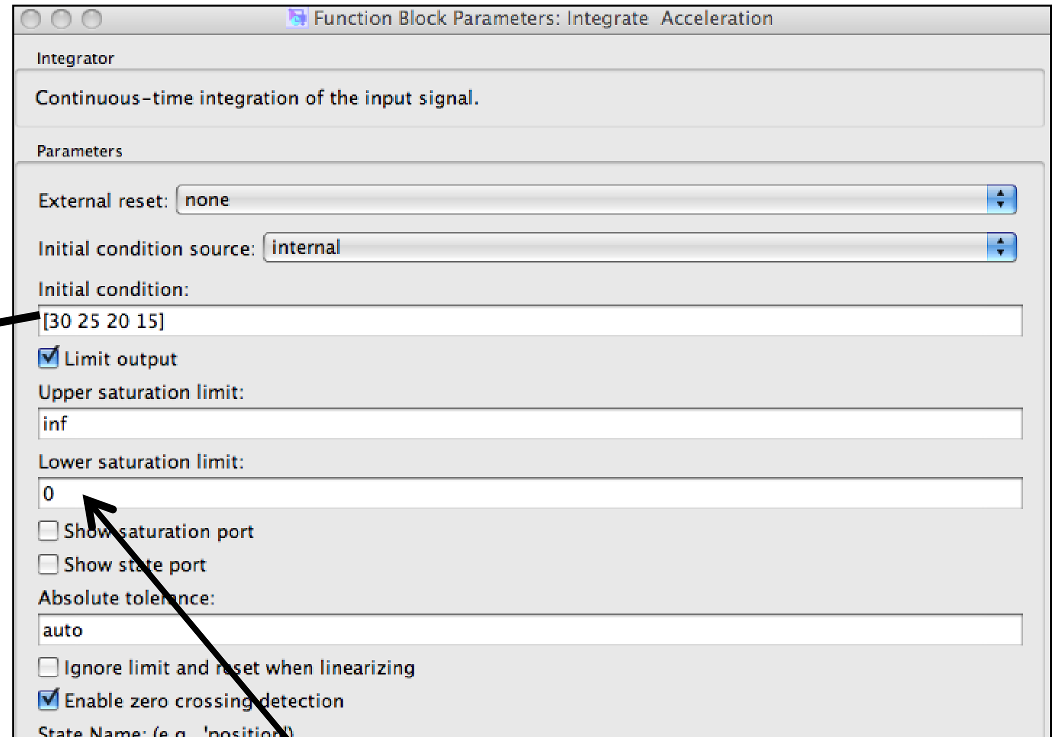


$$a_x = \frac{dV}{dt} = -g\mu$$

Model Initial Conditions

- Aircraft speed = [30 25 20 15] vector of initial speeds





Note: Specify the lower saturation limit to zero
This way speed is never negative

Model Initial Conditions

- Aircraft position = 0 meters at t = 0 (time = 0)

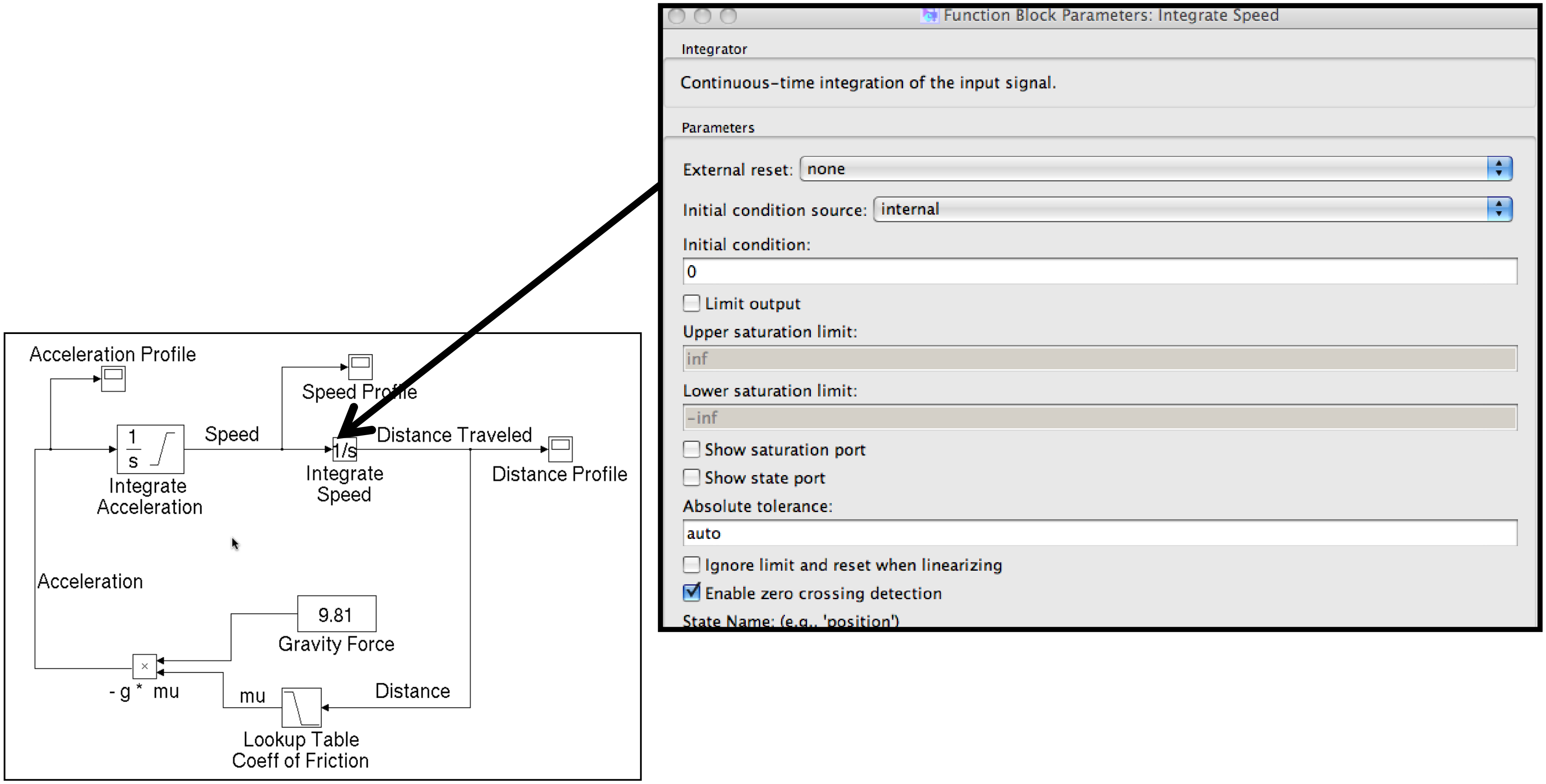
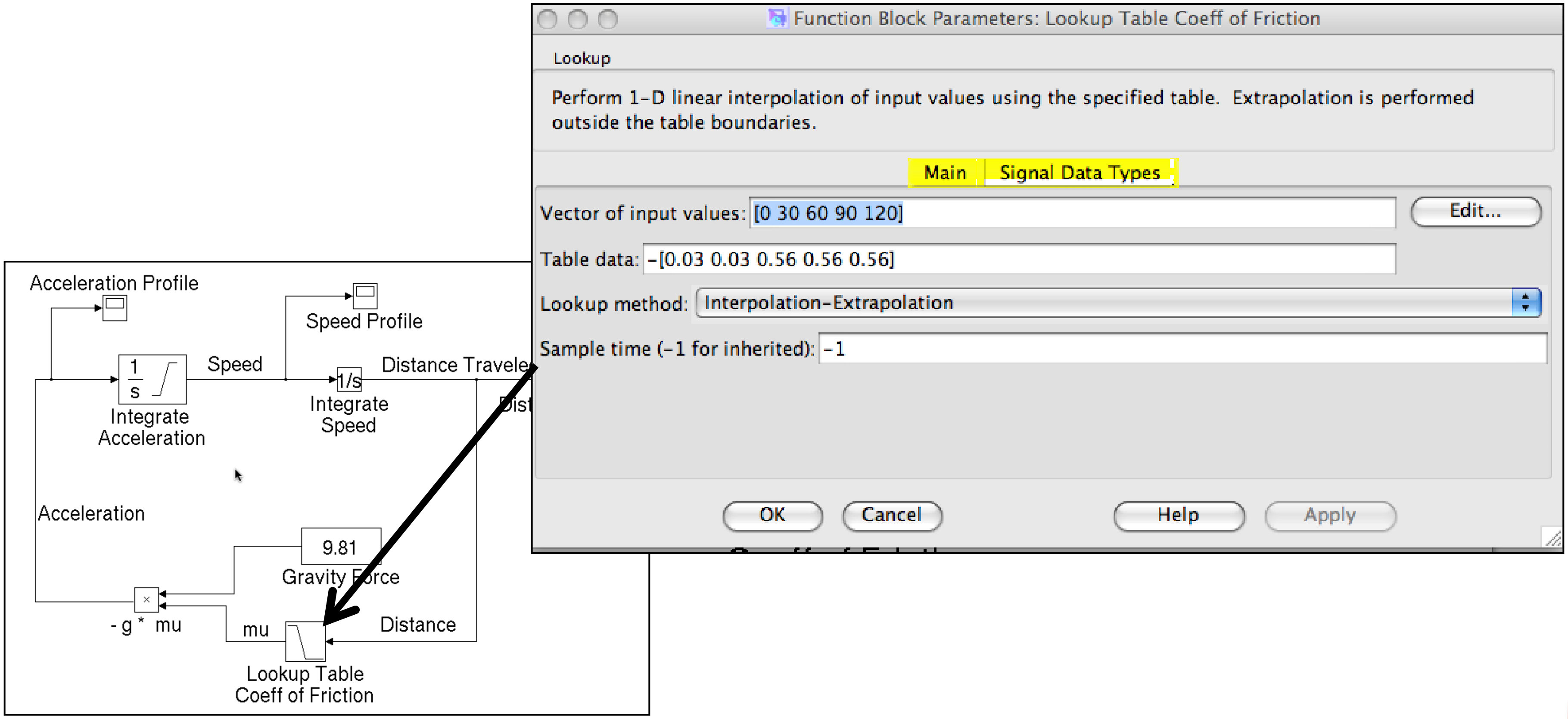
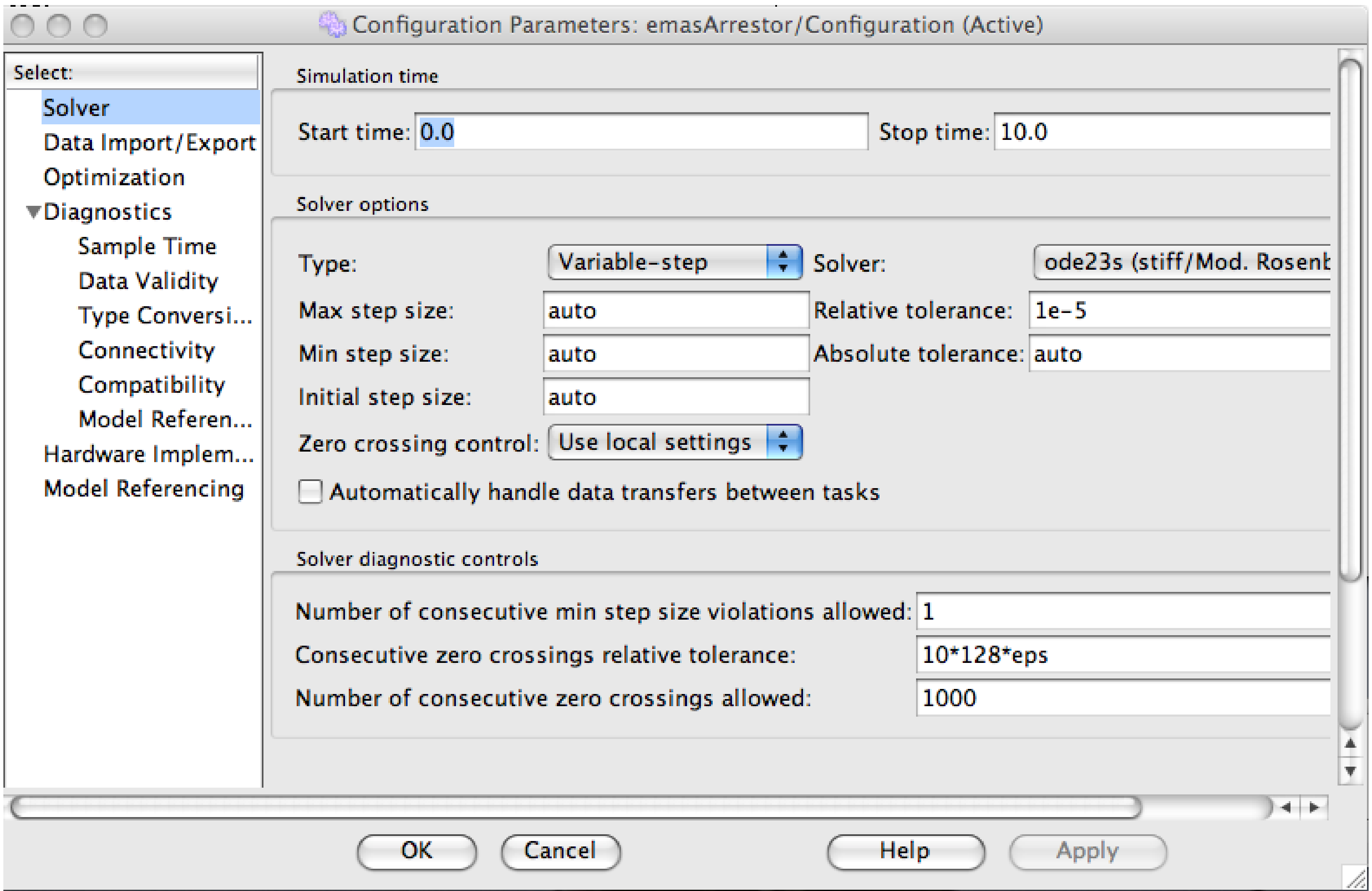


Table Lookup Function

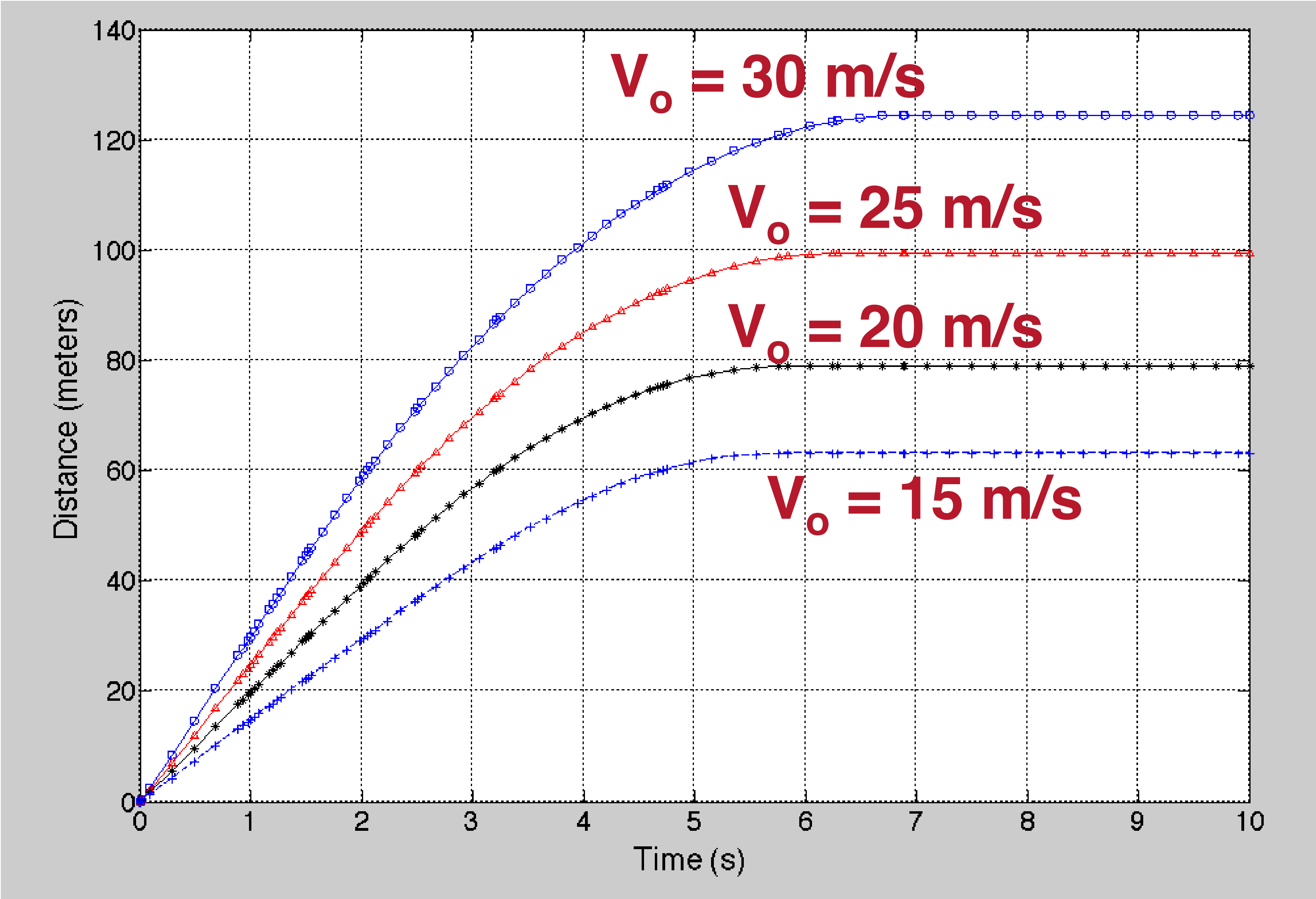
- Specifies the values of (μ) vs distance to simulate the arrestor bed



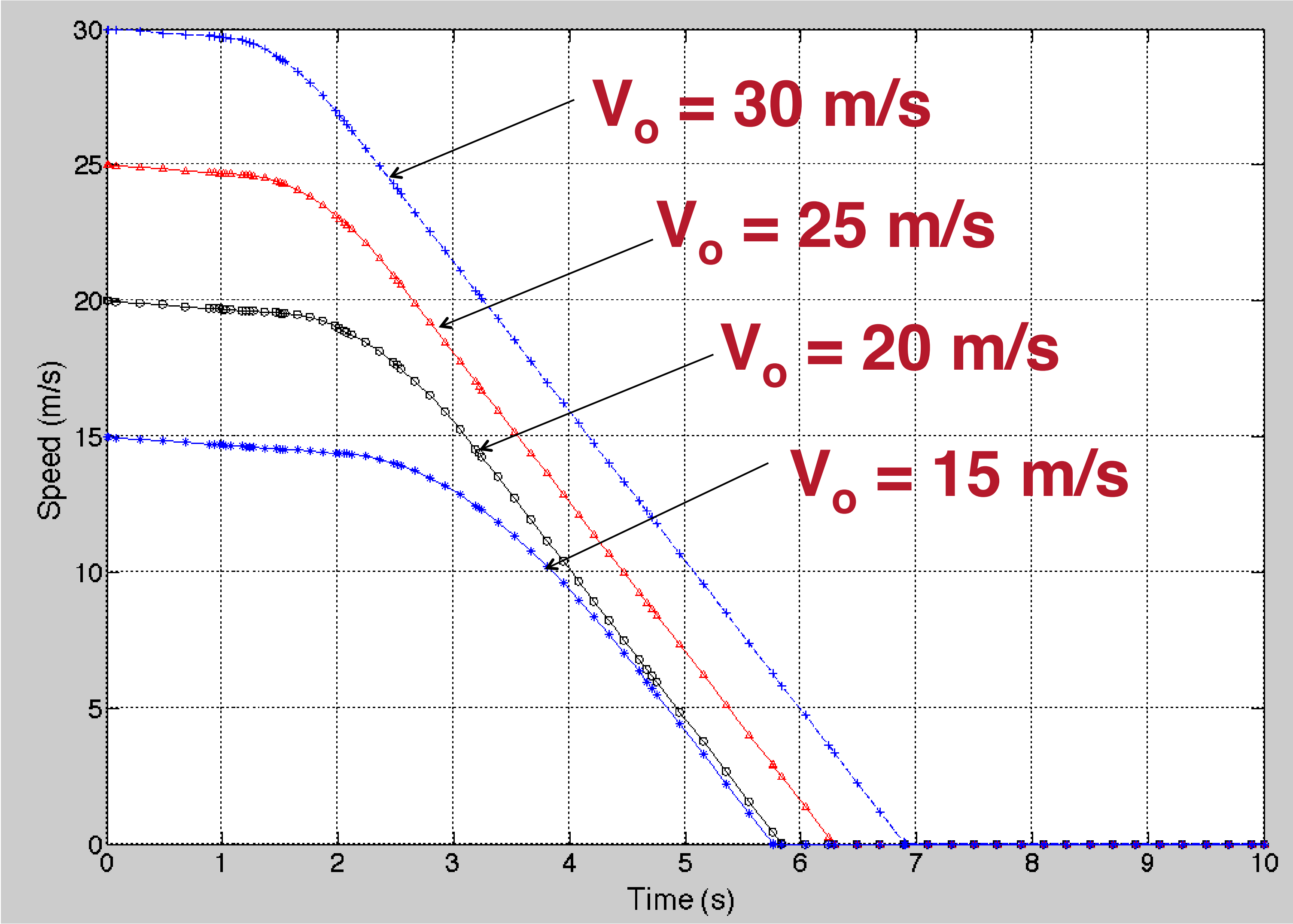
Setup the Configuration to Run



Results (Distance Traveled)



Results (Speed Profiles)



Summary of Results

- The arrestor bed needs to be 125 meters long to stop a medium size transport aircraft if the runway exit speed is set to 70 knots (36 m/s)
- The current FAA design for medium transports is about 400 feet (122 meters) for design exit speeds of 70 knots
- Simple engineering calculations and assumptions allow us to predict complex mechanical behaviors