

Assignment 10: Functions and Differential Equations (40 Points)

Solution

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Problem 1

Steel is an important material to civil engineers because it is used in the construction of building structures. The certification of steel beams requires heating over a period of time to test their properties for fire safety. The first-order differential equation (dT/dt) that estimates the rate of change of temperature of a steel beam as a function of time is:

$$\frac{dT}{dt} = -H1(T - T_a)^{1.2} - H2(T)^{0.25}$$

Where:

$\frac{dT}{dt}$ is the rate of change of the steel beam temperature with time (degrees C./minute)

T is the steel beam temperature (degrees C.)

T_a is the ambient temperature (degrees C.)

$H1, H2$ are a constants of proportionality (1/minutes)

t is the time of the cooling process (minutes)

Task 1

Create a Simulink model to estimate the temperature profile of the steel beam tested in the laboratory. Assume the following parameters:

$T_a = 18$ degrees Celsius

$T_o = 280$ degrees Celsius (initial temperature of the steel beam)

$H1 = 0.08$ (1/minutes)

$H2 = 0.03$ (1/minutes)

Simulate the cooling period to be 60 minutes.

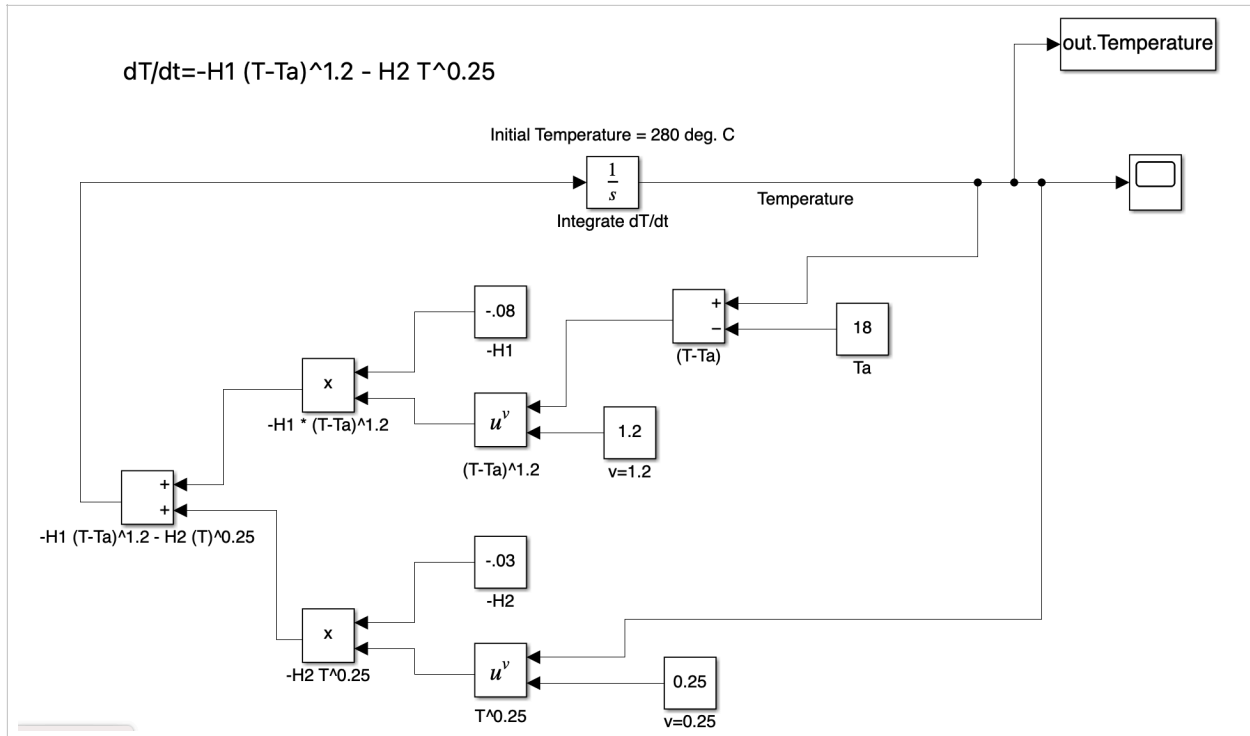


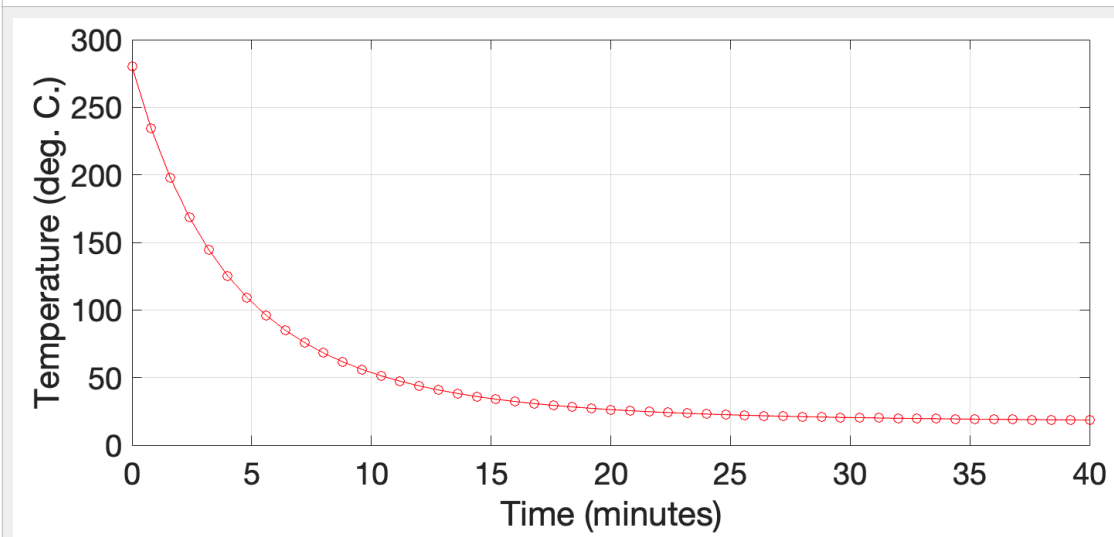
Figure. Simulink model to simulate the temperature profile of a cooling beam.

Task 2

Export the temperature profile to Matlab and make a nice plot of temperature against time.

Use the plot generated to find the time for the beam to reach 200 and 100 degrees. You can find the times from the graphical solution or by interpolating the exact value of time from the Simulink model output.

```
plot_coolingBeamSimulinkModel.m * x +
1 % Script to plot the temperature profile of a cooling beam
2
3 % Define two vectors : time and temperature
4 time = out.Temperature.Time;
5 temperature = out.Temperature.Data;
6
7 % Make the plot
8 plot(time,temperature,'o-r')
9 xlabel('Time (minutes)')
10 ylabel('Temperature (deg. C.)')
11 grid
12
13 % Interpolate to obtain times for temperatures 200 and 100 deg.C.
14 clc
15 timeTo200C = interp1(temperature,time,200);
16 disp(['Time to reach 200 deg. C. is ', num2str(timeTo200C), ' minutes'])
17
18 timeTo100C = interp1(temperature,time,100);
19 disp(['Time to reach 100 deg. C. is ', num2str(timeTo100C), ' minutes'])
```

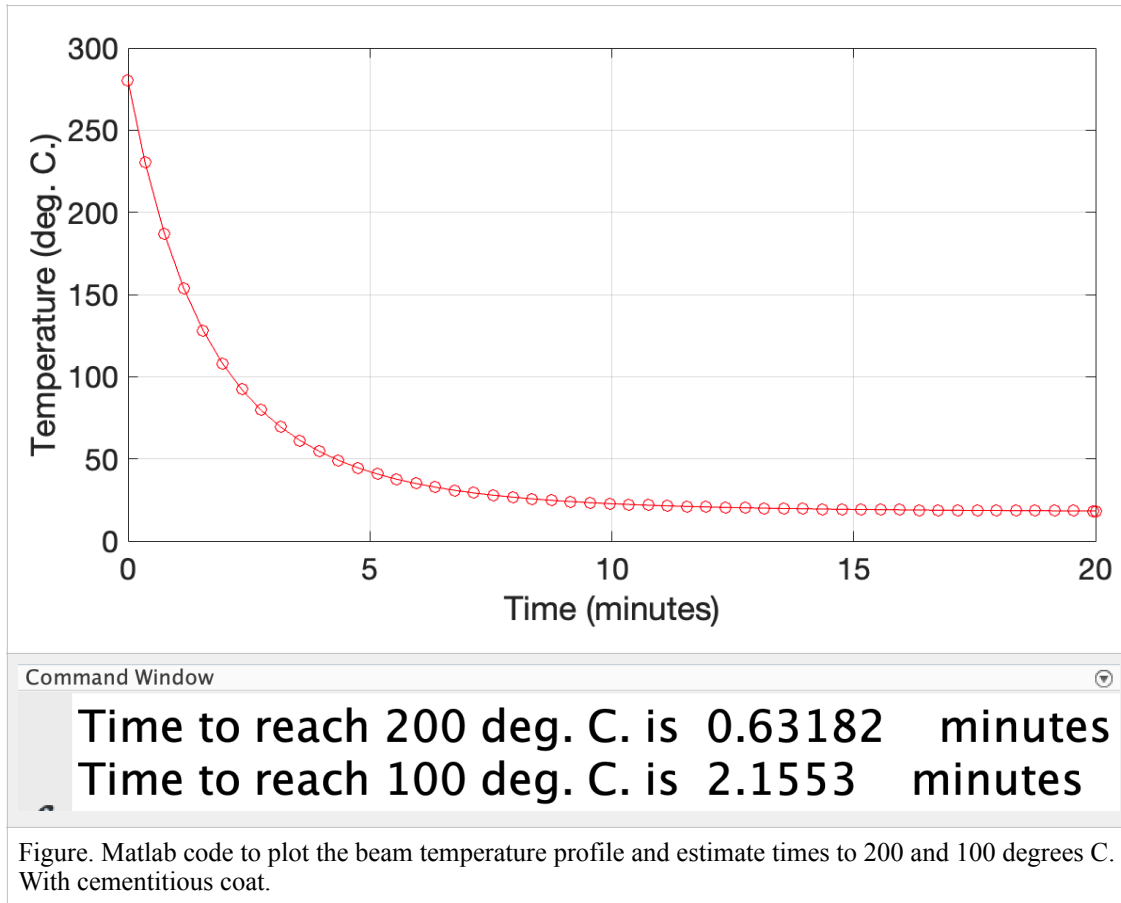


```
Command Window
Time to reach 200 deg. C. is 1.5529 minutes
Time to reach 100 deg. C. is 5.3635 minutes
```

Figure. Matlab code to plot the beam temperature profile and estimate times to 200 and 100 degrees C.

Task 3

If a fire-rated cementitious compound is applied to the steel beam changes the value of H_1 to 0.20, find out the changes in the cooling time to reach 200 and 100 degrees Celsius.



Problem 3

A formula to estimate the roadway noise of an interstate highway is:

$$L = 10 \cdot \log_{10}(q) + 20 \cdot \log_{10}(u) - 10 \cdot \log_{10}(d) + 25;$$

where:

- L = Sound pressure level (dBA) at location of the observer
- q = traffic volume(veh/hr)
- u = mean speed(mi/hr), and
- d = distance from the highway to the center of observer(ft).

Note \log_{10} in Matlab is the logarithm base 10.

Task 1

Write a **Matlab function** to calculate the value of L given values of q, u, and d.

```

noiseFunction.m * x +
1   % function L = noise(q,u,d)
2
3   % Function to calculate the mean noise level of a highway (L)
4   % given:
5   % q = traffic volume(veh/hr)
6   % u = mean speed(mi/hr), and
7   % d = distance from the highway to the center of observer(ft).
8
9   function [L] = noiseFunction(q,u,d)
10
11  L=10*log10(q)+20*log10(u)-10*log10(d)+25;

```

Figure. Matlab function to calculate the noise level generated by the highway.

Task 2:

Create a Matlab script (separate to the function) to test the Matlab script created in Task 1 to calculate the value of L under the following conditions: q = 2500 veh/hr, u = 60 mi/hr, and d = 200 feet. Display the solution in the Command Window. Label appropriately.

```

noiseFunction.m x callnoiseFunction.m * x +
1   % Script calls the function L = noise(q,u,d)
2
3   % Function to calculate the mean noise level of a highway (L)
4   % given:
5   % q = traffic volume(veh/hr)
6   % u = mean speed(mi/hr), and
7   % d = distance from the highway to the center of observer(ft).
8
9   close all
10
11  q=2500;           % veh/hr
12  u=60;            % speed (mph)
13  d=200;           % distance value (feet)
14  % d=200:50:1000; % distance vector (feet)
15
16  L = noiseFunction(q,u,d);
17
18  plot(d,L,'o-r')
19  xlabel('Distance (feet)')
20  ylabel('Sound pressure level (dBA)')

```

Figure. Matlab script to call the noise level function generated by the highway. Line 16 calls the noise function.

Task 3

Modify the Matlab script to create solutions for L as a function of distance to the observer (to be plotted in the x-axis) ranging from 200 to 1,000 feet (steps of 50 feet). Plot the answers obtained. Use Matlab vector operations and not a FOR loop.

```
1 % Script calls the function L = noise(q,u,d)
2
3 % Function to calculate the mean noise level of a highway (L)
4 % given:
5 % q = traffic volume(veh/hr)
6 % u = mean speed(mi/hr), and
7 % d = distance from the highway to the center of observer(ft).
8
9 close all
10
11 q=2500; % veh/hr
12 u=60; % speed (mph)
13 % d=200; % distance value (feet)
14 d=200:50:1000; % distance vector (feet)
15
16 L = noiseFunction(q,u,d);
17
18 plot(d,L,'o-r')
19 xlabel('Distance (feet)')
20 ylabel('Sound pressure level (dBA)')
```

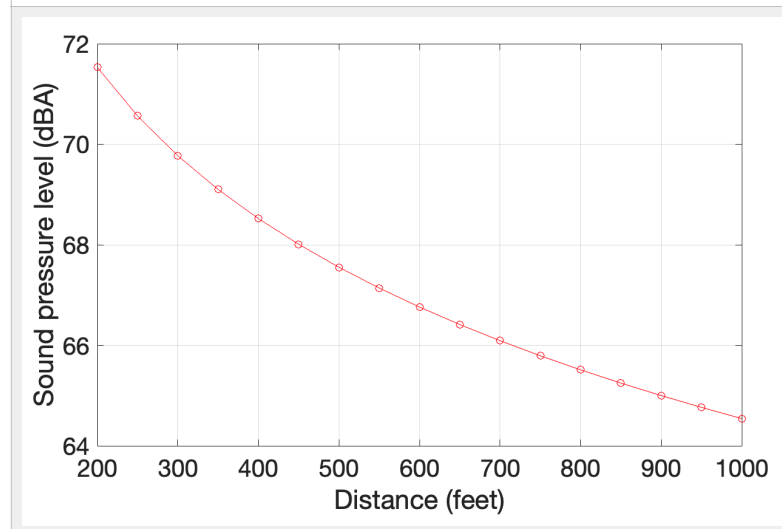


Figure. Matlab script to call the noise level function generated by the highway. Line 16 calls the noise function.

Task 4

OSHA requires parks and recreational areas to be located in land with noise levels (L) below 65 dBA. Using the solution obtained in Task 3, state the minimum distance needed from a park to the highway.

The minimum distance to achieve 65 dBA is 900 feet.

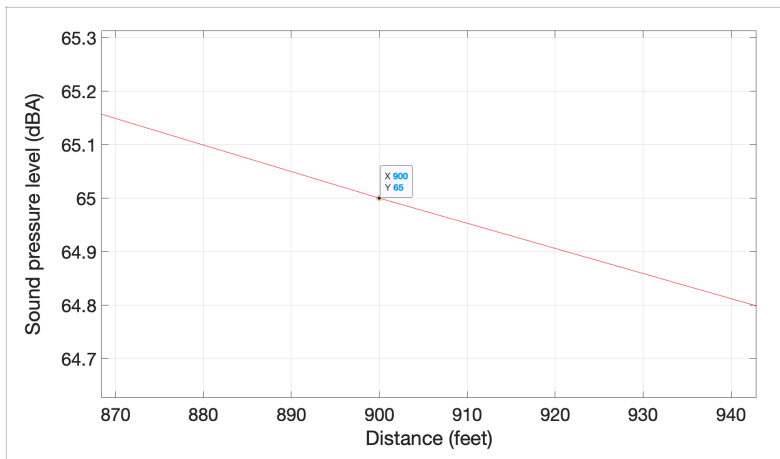


Figure. 65 dBA noise level at 900 feet from the highway.