Assignment 3: Linear Programming

Solution

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Problem 1

Modify the water management pollution control problem described in the class notes and explained in class. New removal costs are presented in Table 1.

Table 1. Removal Costs and Pollution Values for Water Pollution Control Problem.

Source	Removal Cost ($/kg)	Pollution to Lake (kg)
River A	146	27,500
River B	145	21,000
River C	149	24,500
City	215	13,200
Airport	203	18,900

Assume that under a new water mandate by EPA we would like to remove at least 58,000 kg. of the baseline pollution going into the lake. Moreover, airport and city managers want to participate in the pollution removal program by removing at least 60% of their baseline pollution allocations per year. The pollution processing plants at all three rivers need to remove at least a fifth of their pollutants as a minimum according to a new environmental law.

a) Formulate the problem as a linear programming problem. Solve the new problem using Excel Solver and state the optimal cost.

The airport manager would like to invest in a deicing fluid system able to recycle 60% of the pollutants produced by the airport. The new plant is expected to cost $20,000,000 and last for at least 15 years.

b) Using principles of engineering economics and Excel, calculate the yearly payments from the airport authority to a bank to buy the recycling system and pay it off at the end of 15 years. Assume the bank charges 5% yearly over the loan period.

c) I assume that airport operations increase at a rate of 2% per year for the next 15 years. Is the investment in the recycling plant? Comment.

Problem 2

You are in charge of a civil engineering construction company that makes concrete for various highway projects in the State of North Carolina. Your company has various sites across the state to take sand and gravel materials necessary to make a concrete mix. For a construction job near greensboro there are two sites to extract sand and gravel raw materials: a) Sandy Ridge and b) Triad Park. Due to variations in the soil properties at each site, the raw material from Sandy Ridge produces 35% sand and 65% gravel. Triad Park produces 48% sand and 52% gravel.

The construction job in Greensboro requires a minimum of 36,500 cubic meters of sand and gravel mix. The pavement design engineer requires a minimum of 14,900 cubic meters of sand and no more than 19,000 cubic meters of gravel in making the concrete mix for this highway job. The unit delivery costs (includes the cost or raw materials and the hauling costs) are $745 and $820 per cubic meter from Sandy Ridge and Triad Park, respectively.

For each item below, use screen captures to show me how is that the analysis is done.

a) Formulate this problem as a linear programming problem. Clearly state the objective function and the constraint equations of the problem.

Let $x_1 =$ amount of material from Sandy Ridge
$x_2 =$ amount of material from Triad Park

Objective function

$$\text{Minimize } Z = 745x_1 + 820x_2$$

Constraints

$$0.35x_1 + 0.48x_2 > 14900$$

$$0.65x_1 + 0.52x_2 < 19000$$

$$x_1 + x_2 > 36500$$

b) Solve the problem graphically. Plot the lines of constant values of the objective function and show the optimal solution in your plot.

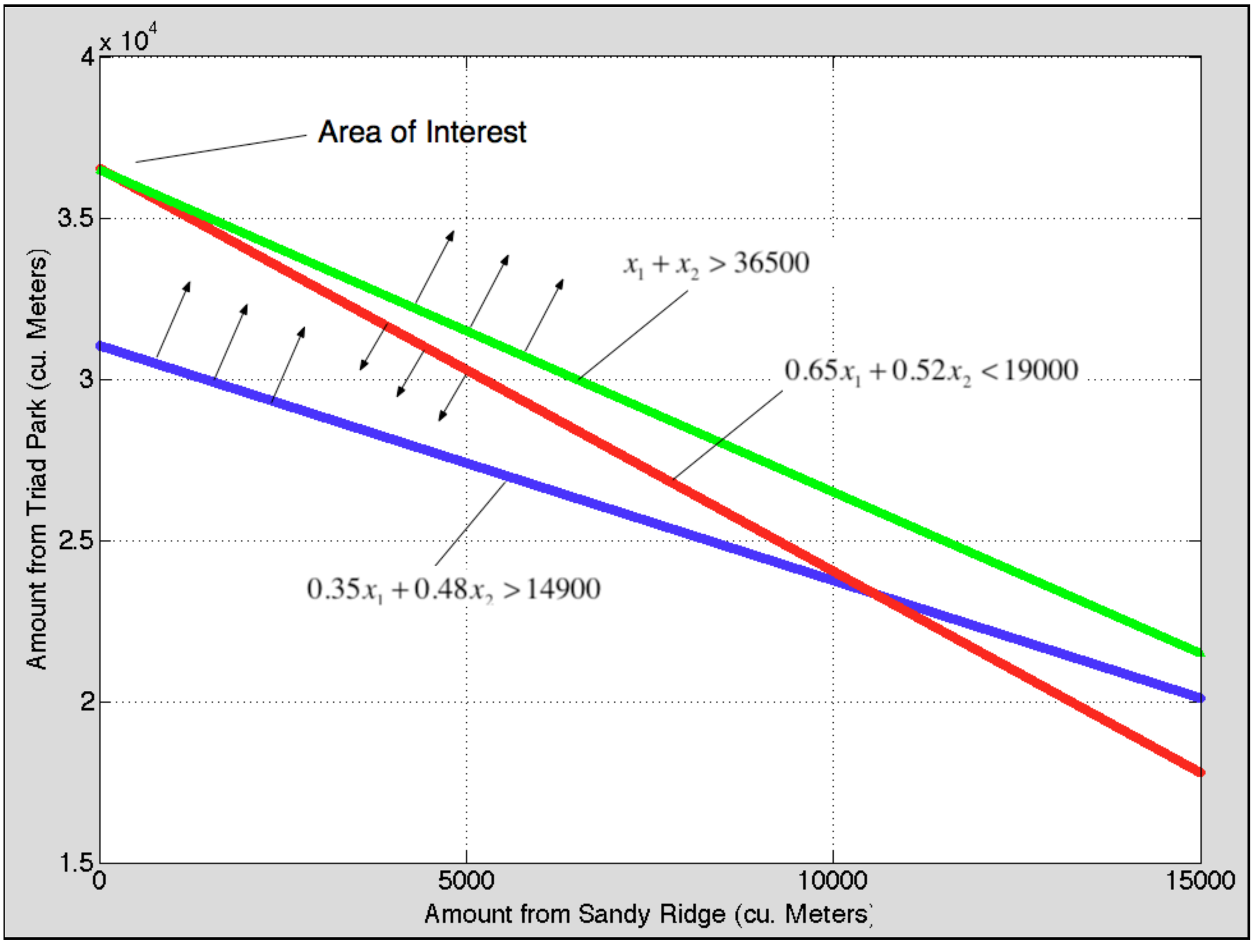


Figure 1. Constraint Equations.

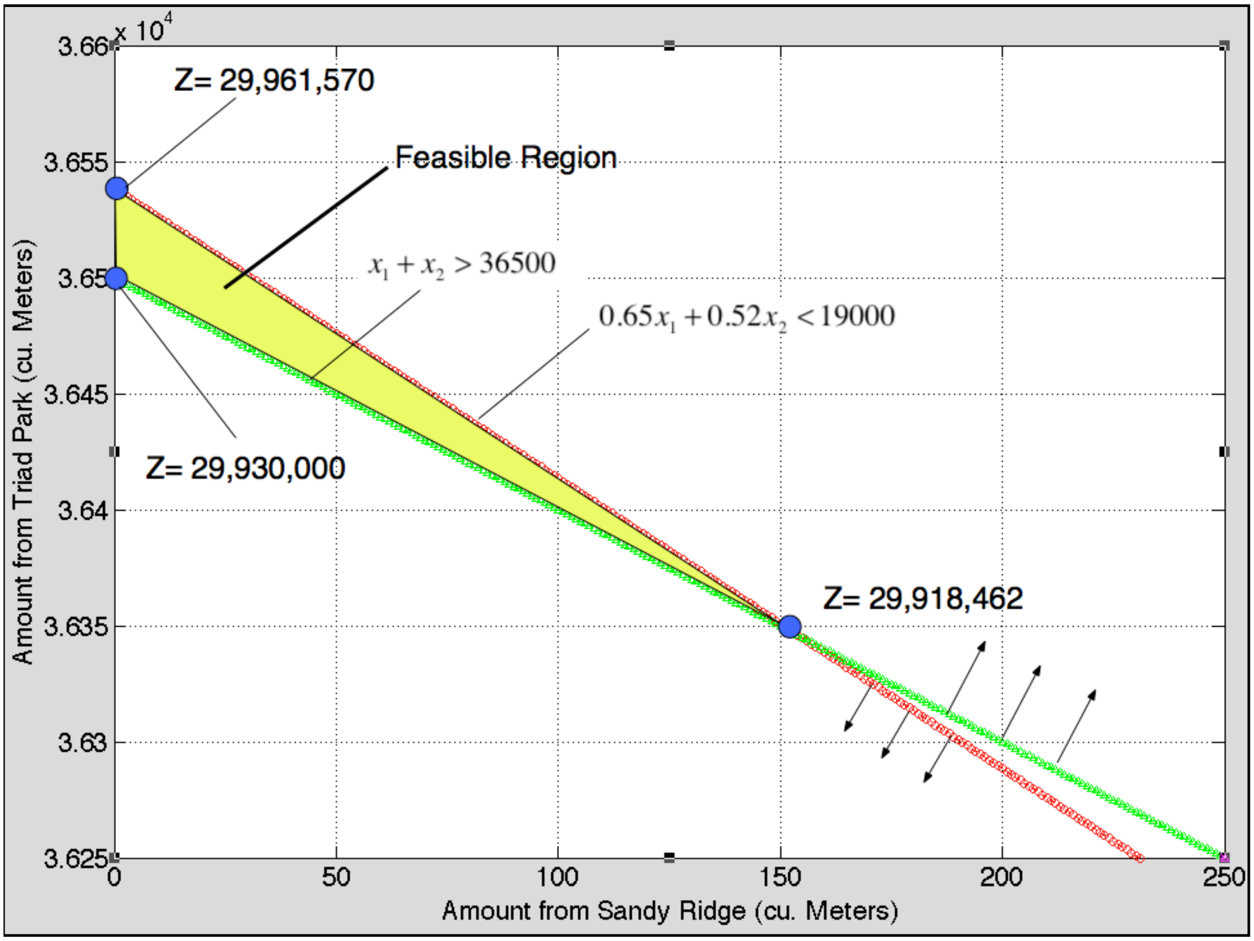


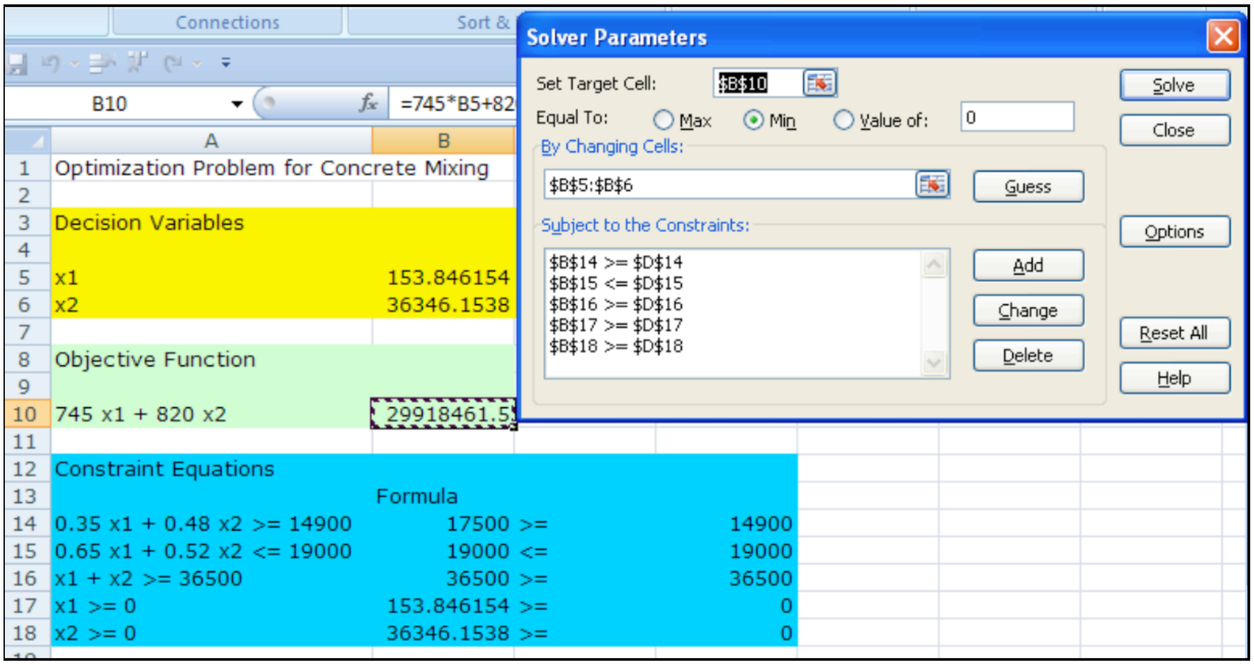
Figure 2. Detail of Feasible Region.

The optimal solution to minimize the value of Z is:

$x_1 =$ 153.85 cu. meters of material from Sandy Ridge

$x_2 =$ 36,346.15 cu. meters of material from Triad Park

c) Setup and solve the problem using Excel Solver.



Optimization Problem for Concrete Mixing			
Decision Variables			
x1	153.846154		Material from Sandy Ridge
x2	36346.1538		Material from Triad Park
Objective Function			
745 x1 + 820 x2	29918461.5		
Constraint Equations			
	Formula		
0.35 x1 + 0.48 x2 >= 14900	17500	>=	14900
0.65 x1 + 0.52 x2 <= 19000	19000	<=	19000
x1 + x2 >= 36500	36500	>=	36500
x1 >= 0	153.846154	>=	0
x2 >= 0	36346.1538	>=	0

Figure 3. Verification of Optimal Solution Using Solver.

Problem 3

A colleague of yours started solving a linear programming. She created the following feasible region plot for this problem.

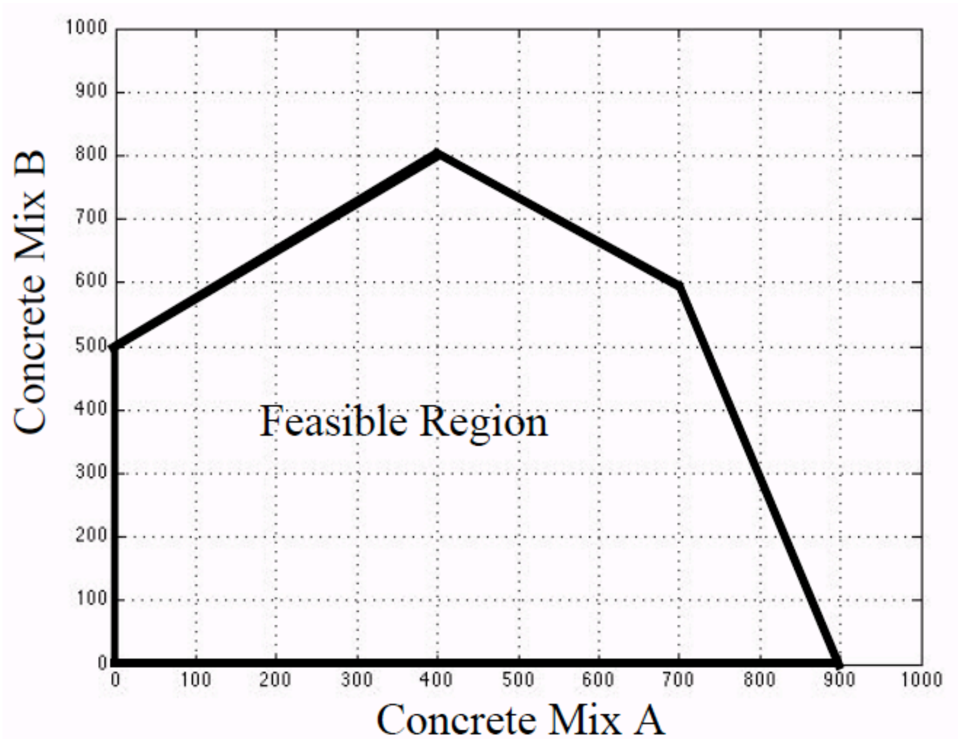


Figure 4. Revenue Production for ACME Concrete Company. Units of Axes are Metric Tons.

The ACME company makes $1600 for every metric ton of concrete mix of type A delivered. The company makes $1735 for every metric ton of concrete mix of type B delivered.

a) Formulate this optimization problem to maximize the revenue for the ACME company. Write down the objective function and the constraints equations.

Let $x_1 =$ be the amount produced of type A
$x_2 =$ be the amount produced of type B

Objective function

$$\text{Maximize } Z = 1600x_1 + 1735x_2$$

Constraints

$$-3/4x_1 + x_2 \leq 500$$

$$2/3x_1 + x_2 \leq 1{,}067$$

$$3x_1 + x_2 \leq 2700$$

Transform the problem to canonical form by adding slack variables to change inequality constraints to equality constraints.

$$Z - 1600x_1 - 1735x_2 = 0$$

$$-3/4x_1 + x_2 + x_3 = 500$$
$$2/3x_1 + x_2 + x_4 = 1{,}067$$
$$3x_1 + x_2 + x_5 = 2700$$

b) Create the first three tables of the **Simplex method** to solve the problem.

Table 1. Initial Table of the Problem. Current Solution is: x_1, x_2, x_3, x_4, x_5 = [0 0 500 1067 2700]. Basic variables are x_3, x_4, x_5. Non-basic variables (i.e., those that are zero in the solution) are x_1, x_2.

Basic Variable	Z	x_1	x_2	x_3	x_4	x_5	RHS
	1	-1600	-1735	0	0	0	0
x_3	0	-3/4	1	1	0	0	500
x_4	0	2/3	1	0	1	0	1067
x_5	0	3	1	0	0	1	2700

Step 1. Identify the maximum gain by introducing variable x_2 into the solution. Variable x_2 has the largest negative coefficient in current table and thus becomes the pivot column.

Step 2. Find the basic variable among x_3, x_4, x_5 that is driven to zero first if x_2 is introduced into the solution. Take the ratio test.

Basic Variable	Z	x_1	x_2	x_3	x_4	x_5	RHS	Ratio
	1	-1600	-1735	0	0	0	0	
x_3	0	-3/4	1	1	0	0	500	500
x_4	0	2/3	1	0	1	0	1067	1067
x_5	0	3	1	0	0	1	2700	2700

The minimum ratio is 500. Therefore, basic variable x_3 is first driven to zero as x_2 increases. select row containing basic variable x_3 as the pivot row to do row operations.

Step 3. Perform row operations to eliminate the coefficient (-1735) in the z-row. In this case I multiply row containing basic variable x_3 by 1735 and add to the z-row.

Basic Variable	Z	x_1	x_2	x_3	x_4	x_5	RHS
	1	-298.75	0	1735	0	0	867,500
x_3	0	-3/4	1	1	0	0	500
x_4	0	2/3	1	0	1	0	1067
x_5	0	3	1	0	0	1	2700

Step 4. Perform row operations in every constraint equation to eliminate the coefficients in pivot column (column containing x_2). Row containing the leaving basic variable x_3 stays the same.

4.1) To eliminate coefficient 1 at the intersection of pivot row containing x_4 and pivot column x_2 multiply the second row by -1 and add to row 3.

Basic Variable	Z	x_1	x_2	x_3	x_4	x_5	RHS
	1	-298.75	0	1735	0	0	867,500
x_3	0	-3/4	1	1	0	0	500
x_4	0	1.41267	0	-1	1	0	567
x_5	0	3	1	0	0	1	2700

4.2) To eliminate coefficient 1 at the intersection of pivot row containing x_5 and pivot column x_2 multiply the second row by -1 and add to row 4. This completes the second Table. Read off Table 2 the values of x_1, x_2, x_3, x_4, x_5. Note that in the new table, x_2 has replaced x_3 in the solution.

Table 2. Current Solution is: x_1, x_2, x_3, x_4, x_5 = [0 500 0 567 2200]. Basic variables are x_2, x_4, x_5. Non-basic variables (i.e., those that are zero in the solution) are x_1, x_3.

Basic Variable	**Z**	x_1	x_2	x_3	x_4	x_5	**RHS**
	1	-298.75	0	1735	0	0	867,500
x_2	0	-3/4	1	1	0	0	500
x_4	0	1.41267	0	-1	1	0	567
x_5	0	3.667	0	-1	0	1	2200

This table is not optimal because the coefficient of variable x_1 is negative in the Z-row. This means, the objective function can be improved if variable x_1 is introduced to the solution. Select the column containing x_1 as the pivot column. Perform the previous steps and find the new table.