Assignment 3: Optimization and Excel Solver

Date Due: Solution Instructor: Trani

Show all your work including code and results of your computation in the spreadsheet as screen captures.

Problem 1

A company develops the following Linear Programming problem to minimize the cost of producing two types of steel pins commonly used the construction industry. The objective function is the profit for the company (in dollars per production batch). The company would like to maximize the profit in solving this problem.

Objective Maximize $Z = 60X_1 + 50 X_2$

Subject to

$X_2 + X_1 <= 220$

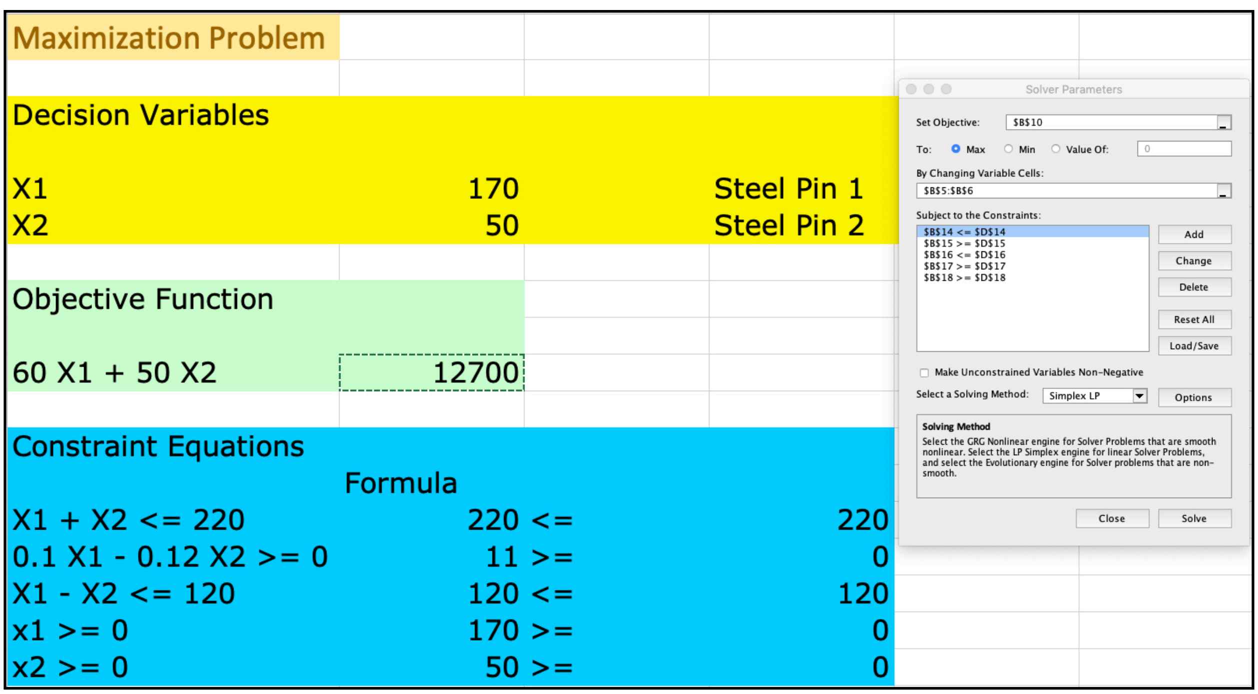
$0.1 X_1 - 0.12 X_2 >= 0$

$X_1 - X_2 <= 120$

$X_1, X_2 >= 0$ (non-negativity conditions)

Task 2

Solve the problem using Excel Solver. State the exact solution found by Excel for the two decision variables. State the value of the objective function for the optimal solution found.

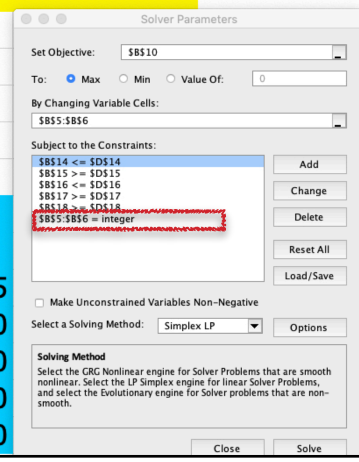


Task 3

Since number of pins to be produced needs to be an integer solution, solve the problem with Excel to obtain an integer solution. State the value of the objective function for the optimal solution found.

The solution is integer for X1 and X2. However, in Solver you can **force an integer solution** by adding another constraint equation in the Solver panel that forces X1 and X2 to be integer (B5 and B6 in my solution). Also, in the "Options" panel select Integer Optimality (%) to zero. The solution shown I changed the first constraint to be <= 220.5 instead of 220 in order to make the optimal solution non-integer. Adding the additional constraint equation to the Solver panel produces the same integer solution as the original problem.

Maximization Problem				
Decision Variables				
X1	170		Steel Pin 1	
X2	50		Steel Pin 2	
Objective Function				
60 X1 + 50 X2	12700			
Constraint Equations				
	Formula			
X1 + X2 <= 220	220	<=	220.5	
0.1 X1 - 0.12 X2 >= 0	11	>=	0	
X1 - X2 <= 120	120	<=	120	
x1 >= 0	170	>=	0	
x2 >= 0	50	>=	0	



Problem 2

You are in charge of a civil engineering pavement company that makes concrete for various highway projects in the State of Virginia. Your company has various sites across the state to take sand and gravel materials necessary to make a concrete mix used in pavement projects. For a construction job near Roanoke, Virginia there are two sites to extract sand and gravel raw materials: a) Starkey and b) Laymantown. Due to variations in the soil properties at each site, the raw material from Starkey produces 43% sand and 57% gravel. Material from Laymantown produces 55% sand and 45% gravel.

The construction job in Roanoke requires a minimum of 85,000 cubic meters of sand and gravel mix. The pavement design engineer requires a minimum of 25,000 cubic meters of sand and no more than 38,000 cubic meters of gravel in making the concrete mix for this highway job. The unit delivery costs (includes the cost of raw materials and the hauling costs) are \$120 and \$130 per cubic meter from Starkey and Laymantown, respectively.

For each task and subtask below, use screen captures to show me how is that the analysis is done.

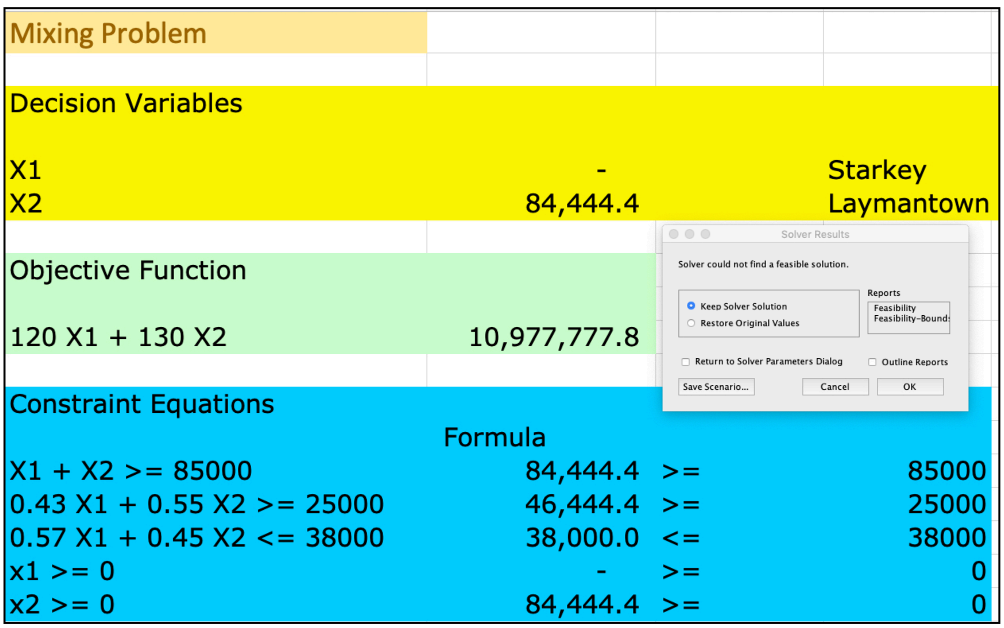
Task 1:

Formulate this problem as a linear programming problem. Clearly state the objective function and the constraint equations of the problem.

Task 2:

Solve the problem graphically. Plot the lines of constant values of the objective function.

Task 3:

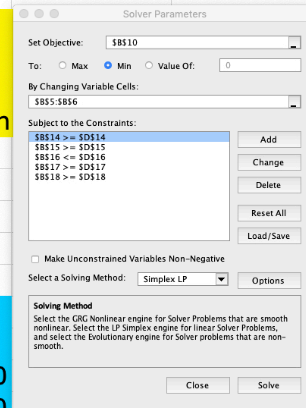


Mixing Problem			
Decision Variables			
X1	-		Starkey
X2	84,444.4		Laymantown
Objective Function			
120 X1 + 130 X2	10,977,777.8		
Constraint Equations			
	Formula		
X1 + X2 >= 85000	84,444.4	>=	85000
0.43 X1 + 0.55 X2 >= 25000	46,444.4	>=	25000
0.57 X1 + 0.45 X2 <= 38000	38,000.0	<=	38000
x1 >= 0	-	>=	0
x2 >= 0	84,444.4	>=	0

Solver cannot find a feasible solution (you can show that graphically as well). However, it offers the closest solution by allocating all the production to the Laymantown site. Note that the solution offered falls short by 556 cubic meters of material.

By relaxing the total material constraint from >=85,000 to say >=83,000 we can find an optimal solution that involves hauling material from both side. The new solution is how below.

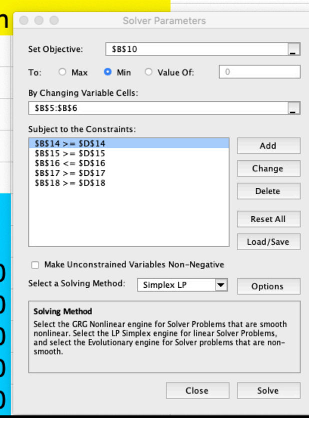
Mixing Problem			
Decision Variables			
X1	5,416.7		Starkey
X2	77,583.3		Laymantown
Objective Function			
120 X1 + 130 X2	10,735,833.3		
Constraint Equations			
	Formula		
X1 + X2 >= 85000	83,000.0	>=	83000
0.43 X1 + 0.55 X2 >= 25000	45,000.0	>=	25000
0.57 X1 + 0.45 X2 <= 38000	38,000.0	<=	38000
x1 >= 0	5,416.7	>=	0
x2 >= 0	77,583.3	>=	0



Task 4:

Suppose that the engineer decides to change the specification of the concrete mix to achieve higher durability against repeated vehicle load cycles. A minimum of 23,000 cubic meters of sand are needed for the job and no less than 51,000 cubic meters of gravel. The solution is shown below. Note that all the allocation is made to site 1 (Starkey).

Mixing Problem			
Decision Variables			
X1	85,000.0		Starkey
X2	-		Laymantown
Objective Function			
120 X1 + 130 X2	10,200,000.0		
Constraint Equations			
	Formula		
X1 + X2 >= 85000	85,000.0	>=	85000
0.43 X1 + 0.55 X2 >= 23000	36,550.0	>=	23000
0.57 X1 + 0.45 X2 <= 51000	48,450.0	<=	51000
x1 >= 0	85,000.0	>=	0
x2 >= 0	-	>=	0



Problem 3

Solve the Osaka Bay problem described in class with the following modifications:

a) Fuji ships carry 700 metric tons of cargo and require a crew of 2.

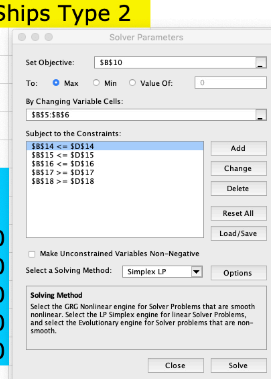
b) Haneda ships carry 1000 metric tons of cargo and require a crew of 3

Task 4:

Solve the problem using Excel Solver. Comment on the results obtained in Tasks 2 and 3.

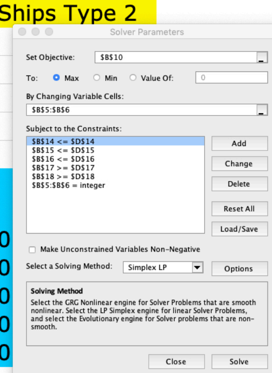
The solution to the revised problem is shown below. Note that the solution, while optimal, does not produce integer values for X1 and X2. Therefore, force the integer solution by adding an additional constraint equation. The integer solution is also presented below.

Revised Problem for Osaka Bay			
Decision Variables			
x1	40.00	Number of Ships Type 1	
x2	33.33	Number of Ships Type 2	
Objective Function			
700 x1 + 1000 x2	61333.33		
Constraint Equations			
	Formula		
2 x1 + 3 x2 <= 180	180.00	<=	180
x1 <= 40	40.00	<=	40
x2 <= 60	33.33	<=	60
x1 >= 0	40.00	>=	0
x2 >= 0	33.33	>=	0



Optimal solution with integer values for X1 and X2 (shown below).

Revised Problem for Osaka Bay			
Decision Variables			
x1	39.00	Number of Ships Type 1	
x2	34.00	Number of Ships Type 2	
Objective Function			
700 x1 + 1000 x2	61300.00		
Constraint Equations			
	Formula		
2 x1 + 3 x2 <= 180	180.00	<=	180
x1 <= 40	39.00	<=	40
x2 <= 60	34.00	<=	60
x1 >= 0	39.00	>=	0
x2 >= 0	34.00	>=	0



Problem 4

Solve the lake pollution control problem described in class with the following attributes:

Pollution Source	Loading (kg/year)	Unit Cost of Removal ($/kg)	Minimum Removal
River A	17,400	36	7,000
River B	16,700	38	8, 000
River C	34,500	32	1/2 of River A removal
Airport	25,600	56	1/2 of River B removal
City	16,500	105 without treatment plant 30 with treatment plant	1/2 of City's original loading
Totals	110,700		

Task 2:

Solve the water pollution control problem if the total desired pollution removal is 45,000 kg. In solving the new problem, assume the city invested in new pollution treatment plant at a cost of $30,000,000. Find out the total cost of pollution removal for this task.

Task 3:

A	B	C	D
Revised Pollution Control Problem			
Variables Names	X1	RiverA	
	X2	RiverB	
	X3	RiverC	
	X4	Airport	
	X5	City	
Decision Variables	X1	7000	
	X2	8000	
	X3	17750	
	X4	4000	
	X5	8250	
Objective	Minimization		
	36X1+38X2+32X3+56X4+105X5	**2,214,250.00**	**$/yr.**
Subject to	X1+X2+X3+X4+X5	45000 >=	45000
	X1<= 17400	7000 <=	17400
	X2<=16700	8000 <=	16700
	X3<=34500	17750 <=	34500
	X4<=25600	4000 <=	25600
	X5<= 16500	8250 <=	16500
	X1>= 7000	7000 >=	7000
	X2>=8000	8000 >=	8000
	X3 - 1/2 X1 >= 0	14250 >=	0
	X4 - 1/2 X2 >= 0	0 >=	0
	X5>= 16500/2	8250 >=	8250

Solution with City cost with no treatment plant

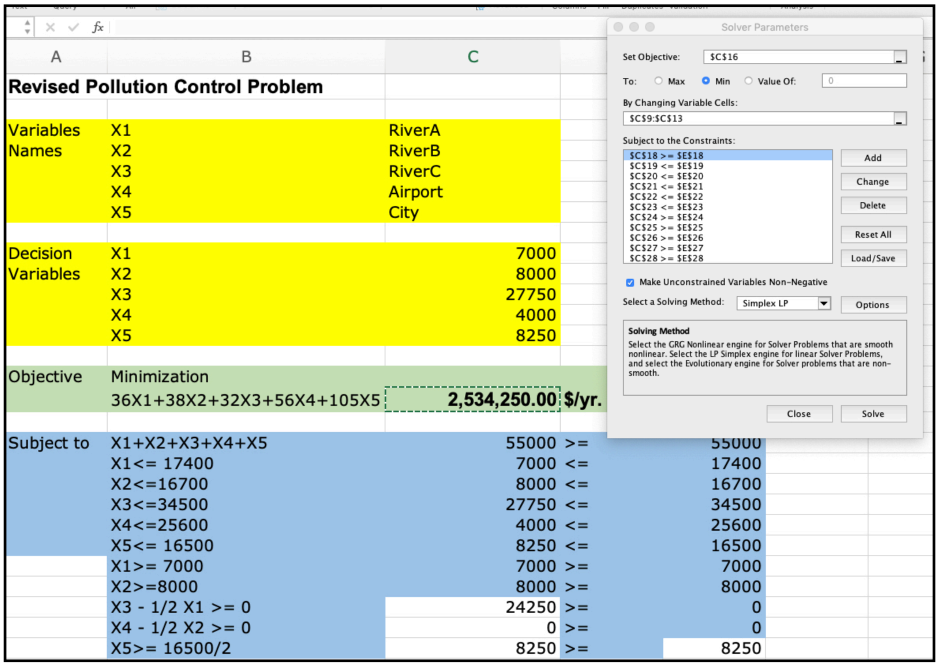
Revised Pollution Control Problem			
Variables Names	X1	RiverA	
	X2	RiverB	
	X3	RiverC	
	X4	Airport	
	X5	City	
Decision Variables	X1	7000	
	X2	8000	
	X3	9500	
	X4	4000	
	X5	16500	
Objective	Minimization		
	36X1+38X2+32X3+56X4+30X5	**1,579,000.00**	**$/yr.**
Subject to	X1+X2+X3+X4+X5	45000 >=	45000
	X1<= 17400	7000 <=	17400
	X2<=16700	8000 <=	16700
	X3<=34500	9500 <=	34500
	X4<=25600	4000 <=	25600
	X5<= 16500	16500 <=	16500
	X1>= 7000	7000 >=	7000
	X2>=8000	8000 >=	8000
	X3 - 1/2 X1 >= 0	6000 >=	0
	X4 - 1/2 X2 >= 0	0 >=	0
	X5>= 16500/2	16500 >=	8250

Solution with City cost with treatment plant

Note that we save 635,000 per year. The treatment plant is $30 million. The payback period is 47 years (assuming no demand increase).

Task 3

Using the solution on Task 2, suppose a new (stricter) environmental law takes effect and It is desired to reduce the total pollution discharge to the lake to 55,000 kg/yr instead. Estimate the cost of removal and the amounts to be removed from each pollution source. Contrast the removal cost in Tasks 2 and 3. Comment.



A	B	C	D	E
Revised Pollution Control Problem				
Variables	X1	RiverA		
Names	X2	RiverB		
	X3	RiverC		
	X4	Airport		
	X5	City		
Decision	X1	7000		
Variables	X2	8000		
	X3	27750		
	X4	4000		
	X5	8250		
Objective	Minimization			
	36X1+38X2+32X3+56X4+105X5	**2,534,250.00**	**$/yr.**	
Subject to	X1+X2+X3+X4+X5	55000	>=	55000
	X1<= 17400	7000	<=	17400
	X2<=16700	8000	<=	16700
	X3<=34500	27750	<=	34500
	X4<=25600	4000	<=	25600
	X5<= 16500	8250	<=	16500
	X1>= 7000	7000	>=	7000
	X2>=8000	8000	>=	8000
	X3 - 1/2 X1 >= 0	24250	>=	0
	X4 - 1/2 X2 >= 0	0	>=	0
	X5>= 16500/2	8250	>=	8250