Assignment 6: Linear Programming

Date Due: February 28 , 2024 Instructor: Trani

Problem 1

Modify the water management pollution control problem described in the class notes and explained in class. New removal costs are presented in Table 1.

Table 1. Removal Costs and Pollution Values for Water Pollution Control Problem.

Source	Removal Cost ($/kg)	Pollution to Lake (kg)
River A	1,400	25,200
River B	1,350	22,300
River C	1,420	18,900
City	1,850	16,700
Airport	1,760	17,800

Assume that under a new water mandate by EPA we would like to remove at least half of the of the baseline pollution into the lake. Moreover, the airport manager would like to participate in the pollution removal program by removing at least 55% of their baseline pollution allocations per year. In order to be socially responsible to communities near the lake, the pollution processing plants at all three rivers need to remove at least one fourth of their pollutants as a minimum according to a new environmental law.

a) Formulate the problem as a linear programming problem. Solve the new problem using Excel Solver and state the optimal cost.

b) The city manager would like to invest in a new processing plant to further reduce pollution into the lake. The new plant is expected to cost $7,600,000 and last for at least 25 years. Using principles of engineering economics and Excel, calculate the yearly payments from the city to a bank to buy the processing plant and pay it off at the end of 25 years. Assume the bank charges 4% yearly over the loan period.

Problem 2

The construction of a new highway requires a minimum of 1,100,000 cubic meters of sand and gravel mixture. The final sand/gravel mixture must contain no less than 572,000 cu. meters of sand (fine aggregate) and no more than 605,000 cu. meters of gravel (coarse aggregate).

The gravel and sand materials can be obtained from two sites: 1) Miramar and 2) San Diego. Table 1 shows the proportions of sand and gravel from each site. Because each site is also used in other construction jobs, the maximum amounts of materials excavated from each site are limited to the following: a) 540,000 cu. meters for Miramar, and 690,000 cu. meters for San Diego.

Table 1. Proportions of Sand and Gravel from the Collection Sites.

Site	Proportion of Sand (%)	Proportion of Gravel (%)
Miramar	52	48
San Diego	43	57

The costs of collection and transportation of a cubic meter of material are: a) $650 for Miramar, $670 for San Diego.

A) Setup the problem as a linear programming problem. The objective is to **minimize the cost of producing the concrete for the highway project**.

B) Use the Simplex method to setup by hand the **first two tables of the problem**. For each table indicate the Basic Variables, Non Basic Variables and the value of the objective function (Z). The first table requires the problem to be in standard form.

C) Find the optimal solution that minimizes the cost using **Excel Solver**. Clearly state the values of the decision variables and the value of the objective function in the optimal solution.

D) If the San Diego site offers a 7% discount in purchases of more than 500,000 cu. meters of material, would you consider their offer and re-allocate differently the procurement of sand and gravel from both sites? Explain and solve using Excel Solver to support your answer.

Problem 3

A company develops a sketch in two dimensions of a Linear Programming problem to minimize the cost of producing two types of commonly used steel rebars (called X_1 and X_2 in the follow up equations) used in the construction industry. The objective of the problem is to maximize the profit for the company (in dollars per production batch). The company would like to maximize the profit in solving this problem.

Objective Maximize $Z = 340\ X_1 + 326\ X_2$

Subject to

$X_2 + 1.1\ X_1 <= 270$

$X_1 + 6X_2 <= 1260$

$3X_1 + X_2 <= 580$

$X_1, X_2 >= 0$ (non-negativity conditions)

For each task below, use screen captures to show your work. Show the formulas of the cells to make out task simpler in grading. Also, show the Solver panel to help in grading.

a) Solve the **problem graphically**. State the optimal solution found for the two decision variables. State the value of the objective function for the optimal solution found. In the graphical solution, label the corner points and state the value of the objective function at each corner point.

b) Solve the **problem manually using the Simplex Method explained in class**. Show all your steps and tableaus. Indicate the Basic Variables (BV) and the Non Basic Variables (NBV) in every tableau. Also highlight the value of the objective function in every tableau.

c) Solve the problem using Excel Solver. State the solution found by Excel for the two decision variables. State the value of the objective function for the optimal solution found. Compare the Excel Solver solution with the solution obtained manually in Task 2.

d) Since number of steel rebars to be produced needs to be an integer solution, solve the problem with Excel to obtain an integer solution. State the value of the objective function for the optimal solution found.