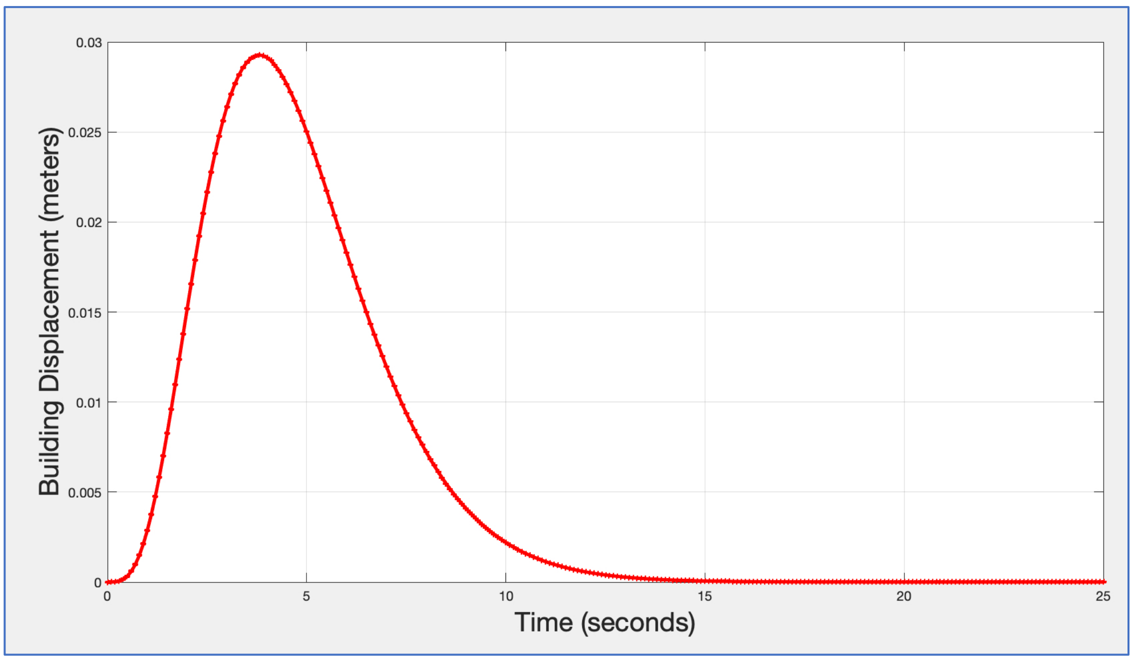
CEE 3804 – Computer Applications in Civil and Environmental Engineering

Assignment 6 Solution

Problem 1

a) Defined time and displacement vectors and plotted a displacement vs time graph

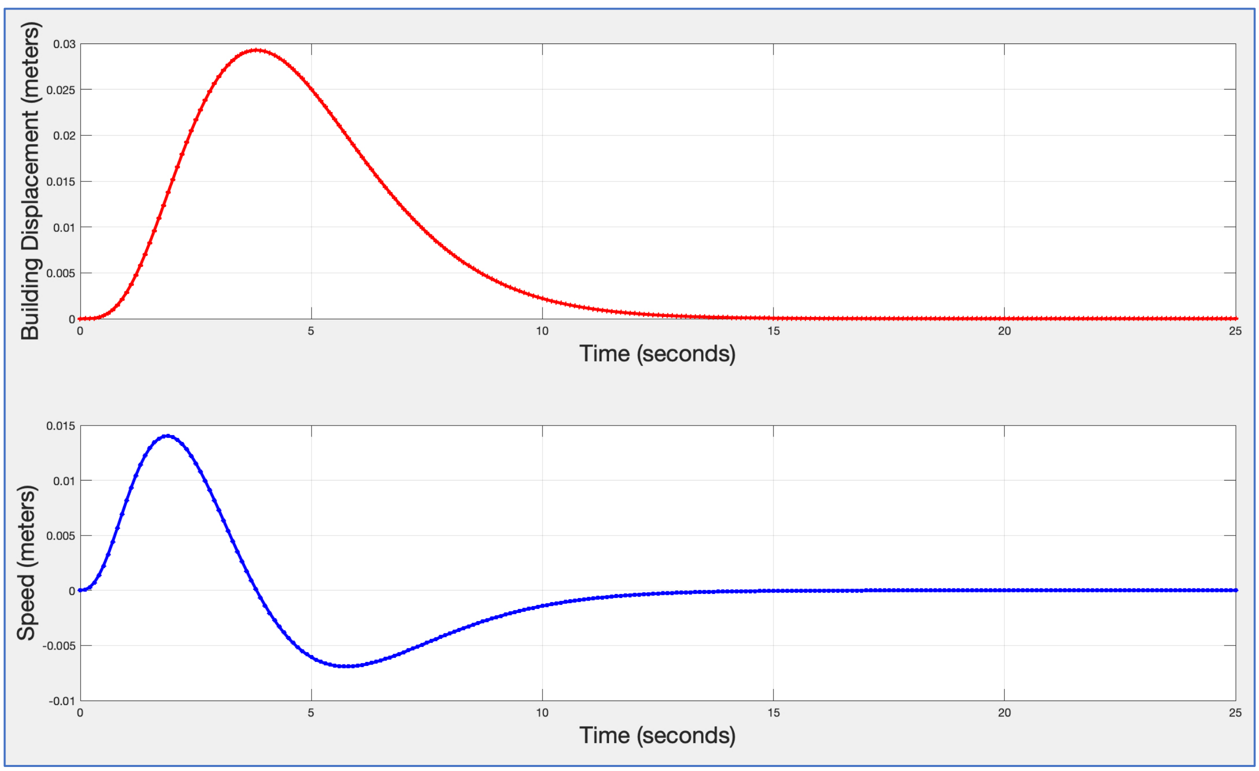
```
% representing the time and horizontal movement
% experienced by a building during an earthquake:

time=0:0.1:25;
x=time.^1.85.*exp(-time).*(1-cos(time/8));

% where:
% time = time in seconds
% x=horizontal displacement of the building in meters

plot(time,x,'o-r',...
            'LineWidth',2.5,...
            'MarkerEdgeColor','r',...
            'MarkerFaceColor','r',...
            'MarkerSize',2)
xlabel('Time (seconds)','fontsize',20)
ylabel('Building Displacement (meters)','fontsize',20)
grid
```



b) In the figure plot, go to Tools > Edit Plot
Select and right click the curve to change the color, line width and marker

c) Created variable z and plotted graphs displacement vs time and velocity vs time in the same window

```
z = gradient(x,time);         % units are m/s

figure
subplot(2,1,1)
plot(time,x,'o-r',...
            'LineWidth',2.5,...
            'MarkerEdgeColor','r',...
            'MarkerFaceColor','r',...
            'MarkerSize',2)
xlabel('Time (seconds)','fontsize',20)
ylabel('Building Displacement (meters)','fontsize',20)
grid

subplot(2,1,2)
plot(time,z,'^-b',...
            'LineWidth',2.5,...
            'MarkerEdgeColor','b',...
            'MarkerFaceColor','b',...
```



d) Select a few points to test if the gradient function is doing its job. The building horizontal speed is the derivative of the displacement as a function of time.

Time (s)	Displacement (m)	Speed (m/s)	Result
0	0	0	Slope is zero (checks)
1.9	0.0138	0.0140	Maximum slope (speed) Inflection point
3.8	0.0293	~0	Slope if flat as expected (checks)

Another way to check is to take the derivative of the function (x = f(t)).

$$x = time^{1.85} * e^{-time} * \left(1 - \cos\left(\frac{time}{8}\right)\right)$$

$$x = \left(time^{1.85} * e^{-time}\right) - \left[time^{1.85} * e^{-time} * \cos\left(\frac{time}{8}\right)\right]$$

Velocity (**using the product rule**),

$$\frac{dx}{dtime}\left(time^{1.85} * e^{-time}\right) = time^{1.85} * \frac{dx}{dtime}\left(e^{-time}\right) + \frac{dx}{dtime}\left(time^{1.85}\right) * e^{-time}$$

$$\frac{dx}{dtime}\left(time^{1.85} * e^{-time}\right) = time^{1.85} * \left(-e^{-time}\right) + 1.85 * time^{(1-1.85)} * e^{-time}$$

$$\frac{dx}{dtime}\left(time^{1.85} * e^{-time} * \cos\left(\frac{time}{8}\right)\right)$$

$$= time^{1.85} * e^{-time} * \frac{dx}{dtime}\left(\cos\left(\frac{time}{8}\right)\right) + time^{1.85} * \frac{dx}{dtime}\left(e^{-time}\right) * \cos\left(\frac{time}{8}\right) + \frac{dx}{dtime}\left(time^{1.85}\right) * e^{-time} * \cos\left(\frac{time}{8}\right)$$

$$= time^{1.85} * e^{-time} * \left(-\frac{\sin\left(time/8\right)}{8}\right) + time^{1.85} * \left(-e^{-time}\right) * \cos\left(\frac{time}{8}\right) + 1.85 * time^{(1-1.85)} * e^{-time} * \cos\left(\frac{time}{8}\right)$$

$$\frac{dx}{dtime}$$

$$= \left[time^{1.85} * \left(-e^{-time}\right) + 1.85 * time^{(1-1.85)} * e^{-time}\right] - \left[time^{1.85} * e^{-time} * \left(-\frac{\sin\left(time/8\right)}{8}\right) + time^{1.85} * \left(-e^{-time}\right) * \cos\left(\frac{time}{8}\right) + 1.85 * time^{(1-1.85)} * e^{-time} * \cos\left(\frac{time}{8}\right)\right]$$

$$= -\left(time^{1.85} * e^{-time}\right) + \left(1.85 * time^{0.85} * e^{-time}\right) + \left(time^{1.85} * e^{-time} * \frac{\sin\left(time/8\right)}{8}\right) + \left(time^{1.85} * e^{-time} * \cos\left(\frac{time}{8}\right)\right) - (1.85 * time^{0.85} * e^{-time} * \cos\left(\frac{time}{8}\right))$$

$$= \left(time^{1.85} * e^{-time}\right) * \left(-1 + \frac{\sin\left(time/8\right)}{8} + \cos\left(\frac{time}{8}\right)\right) + \left(1.85 * time^{0.85} * e^{-time}\right) * (1 - \cos\left(\frac{time}{8}\right))$$

Verification:

***Note that angles are in radians, not degrees**

When time = 5,

$$x = 5^{1.85} * e^{-5} * (1 - \cos\left(\frac{5}{8}\right))$$

$$x = 0.025$$

$$\frac{dx}{dtime} = (5^{1.85} * e^{-5}) * \left(-1 + \frac{\sin\left(5/8\right)}{8} + \cos\left(\frac{5}{8}\right)\right) + (1.85 * 5^{0.85} * e^{-5}) * (1 - \cos\left(\frac{5}{8}\right))$$

$$\frac{dx}{dtime} = -0.006$$

Problem 2

a) Copied data and saved file in matlab

Autobahn_data.m

```
% Traffic Flow Data
%
% Autobahn data
%
% Column 1 = Density (veh/km-lane)
% Column 2 = Speed (km/hr)
0.08     160.00
0.08     152.00
0.00     0.00
0.00     0.00
0.07     162.00
```

b) Reading data using load command

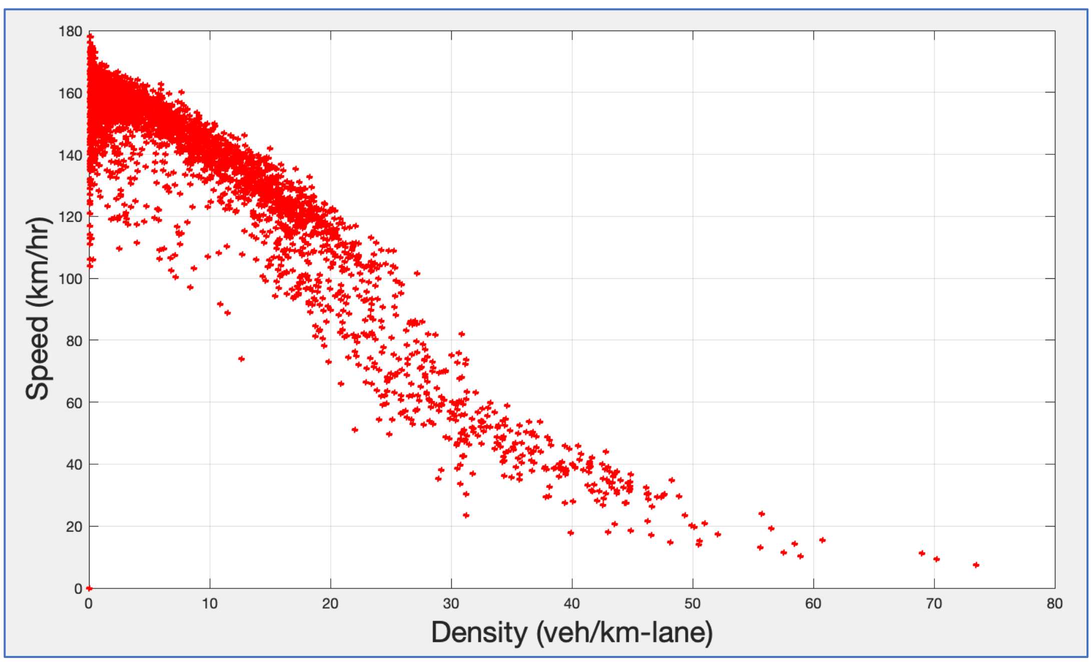
```
% Script to load data from the Autobahn_data file
load Autobahn_data.m;
```

c) Created variables for each column

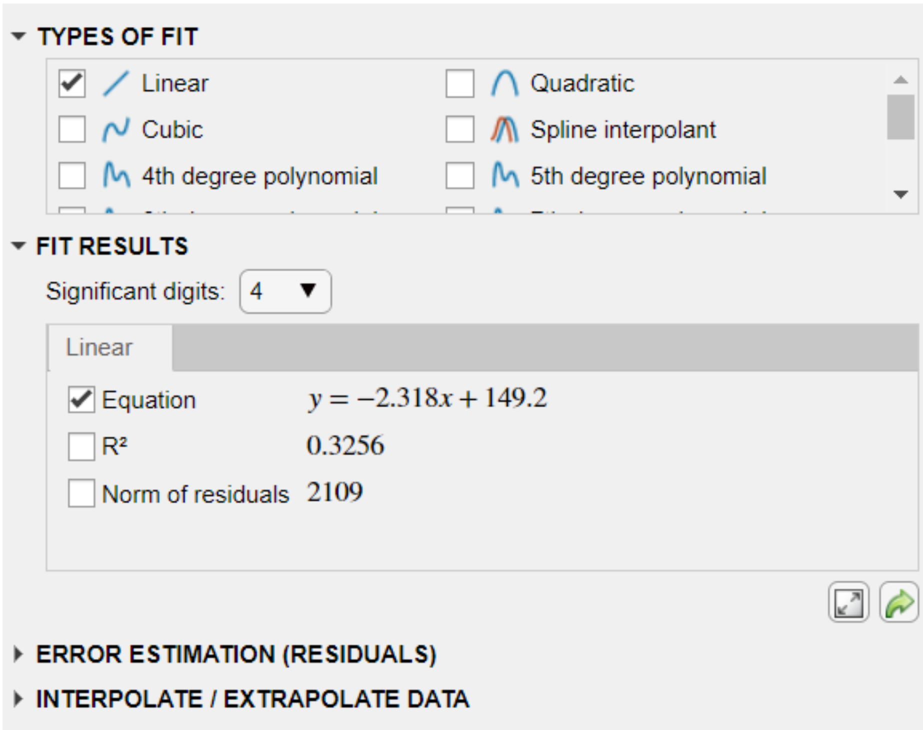
```
% Script to load data from the Autobahn_data file
load Autobahn_data.m;
speed = Autobahn_data(:,2);  % vector with values of speed
density = Autobahn_data(:,1); % vector with values of density
```

d) Traffic density vs speed plot

```
load autobahnData.m

density = autobahnData(:,1);
speed = autobahnData(:,2);

% Make a plot

plot(density,speed,'or',...
        'LineWidth',2.5,...
        'MarkerEdgeColor','r',...
        'MarkerFaceColor','r',...
        'MarkerSize',2)
xlabel('Density (veh/km-lane)','fontsize',20)
ylabel('Speed (km/hr)','fontsize',20)
grid
```



e) Linear regression using Basic Fitting tool



f) Traffic speed,

$$y = -2.318 * 50 + 149.2$$
$$y = 33.3$$

Traffic speed for a density of 50 veh/km is 33.3 km/hr

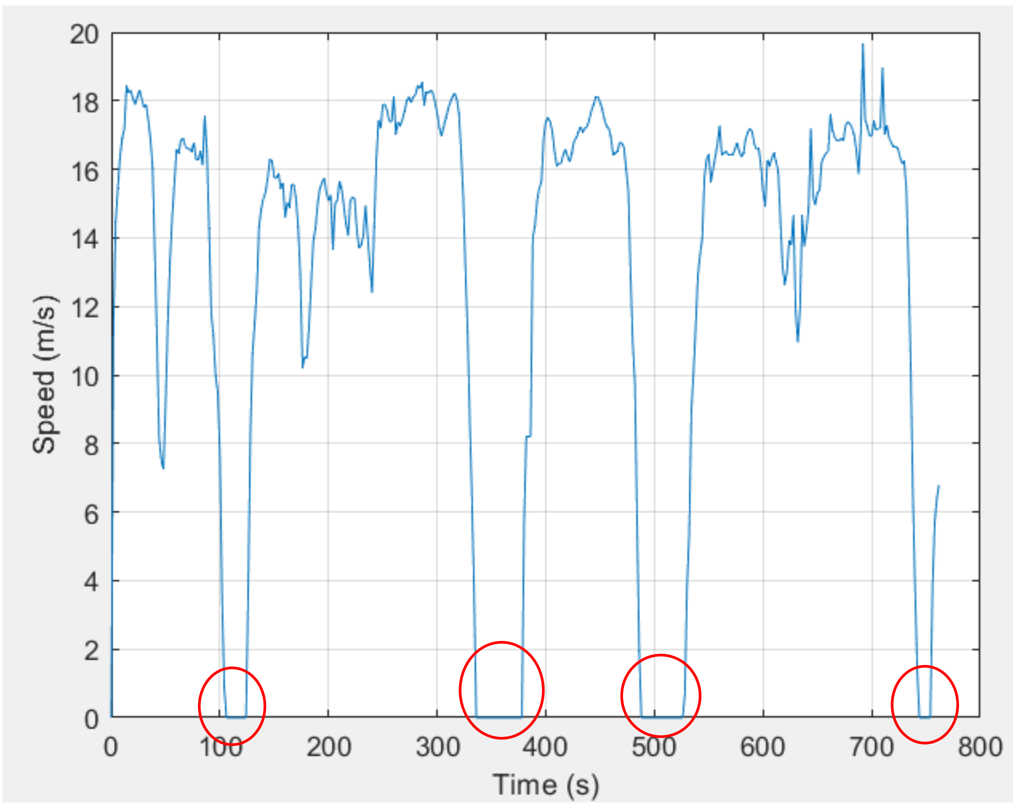
Problem 3

a) Imported GPS data to matlab

```
%Script to load GPS data
load GPS_data.m;
```

b) Time vs Speed

```
%Script to load and plot GPS data
load GPS_data.m;
time = GPS_data(:,1);  % vector with values of time in s
dist = GPS_data(:,2);  % vector with values of distance in m
S = GPS_data(:,3); % vector with values of speed in km/hr
speed = S/3.6; % vector with values of speed in m/s
%Plot time vs speed
plot(time,speed)
xlabel('Time (s)')
ylabel('Speed (m/s)')
grid
```



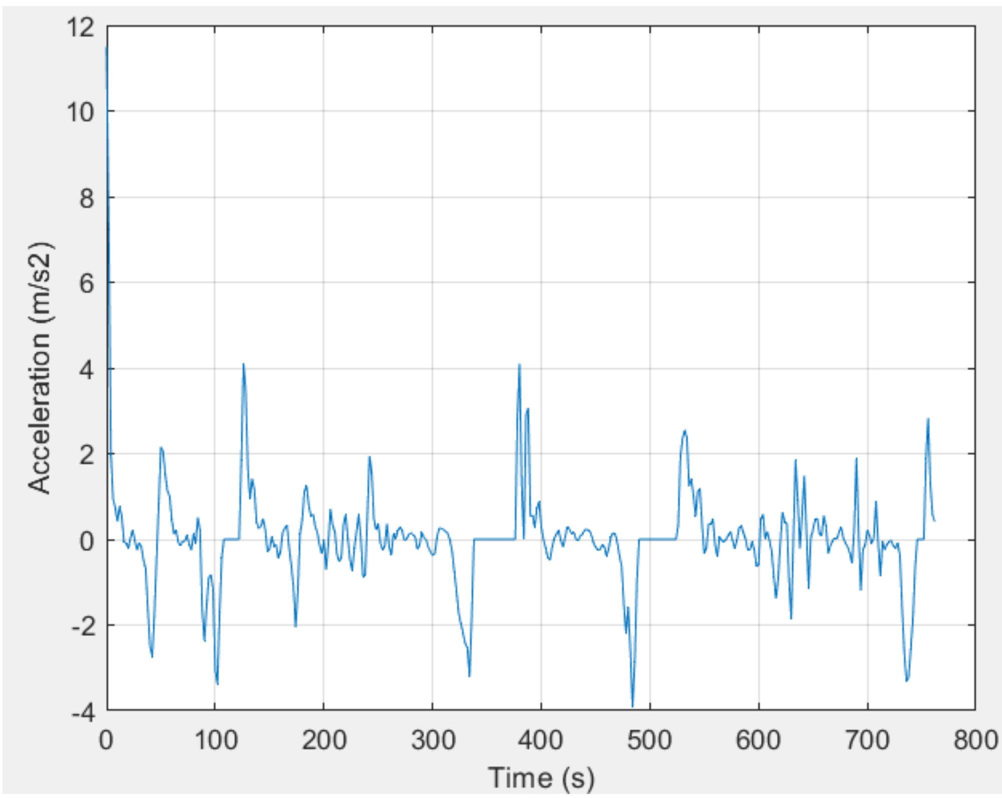
The vehicle makes 4 stops

c) Converted speed from km/hr to m/s

```
S = GPS_data(:,3); % vector with values of speed in km/hr
speed = S/3.6; % vector with values of speed in m/s
```

d) Time vs Acceleration

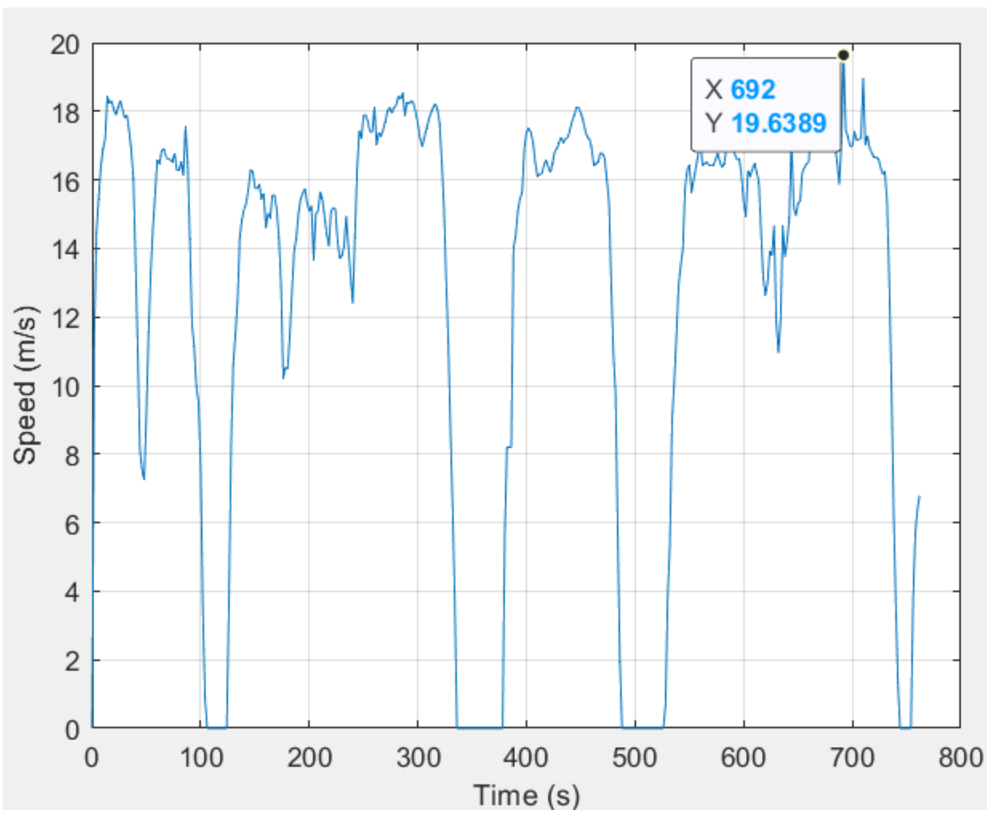
```
%Script to load and plot GPS data
load GPS_data.m;
time = GPS_data(:,1);  % vector with values of time in s
dist = GPS_data(:,2);  % vector with values of distance in m
S = GPS_data(:,3); % vector with values of speed in km/hr
speed = S/3.6; % vector with values of speed in m/s
a = gradient(speed); % vector with values of acceleration in m/s2
%Plot time vs acceleration
plot(time,a)
xlabel('Time (s)')
ylabel('Acceleration (m/s2)')
grid
```



e) Largest speed during the journey = 19.6389 m/s
Time at largest speed = 692 s

```
%Script to load and plot GPS data
load GPS_data.m;
time = GPS_data(:,1);  % vector with values of time in s
dist = GPS_data(:,2);  % vector with values of distance in m
S = GPS_data(:,3); % vector with values of speed in km/hr
speed = S/3.6; % vector with values of speed in m/s
a = gradient(speed); % vector with values of acceleration in m/s2
max_speed = max(speed);
disp(['Largest speed during the journey = ',num2str(max_speed)])
%Plot time vs acceleration
plot(time,speed)
xlabel('Time (s)')
ylabel('Speed (m/s)')
grid
```

```
>> read_GPS_data
Largest speed during the journey = 19.6389
```



f) Average speed of the car for the complete profile is 12.6457 m/s

```
avg_speed = mean(speed); %average speed
disp(['Average speed of the journey = ',num2str(avg_speed)])
```

```
>> read_GPS_data
Average speed of the journey = 12.6457
```

g) Total distance of the journey = 1865010.8 m

```
tot_dist = sum(dist);
disp(['Total distance of the journey = ',num2str(tot_dist)])
```

```
>> read_GPS_data
Total distance of the journey = 1865010.8
```

h) Total number of seconds the car traveled at speeds greater than 30 km/hr is 111192 s

```
load GPS_data.m;
time = GPS_data(:,1);  % vector with values of time in s
dist = GPS_data(:,2);  % vector with values of distance in m
S = GPS_data(:,3); % vector with values of speed in km/hr
speed = S/3.6; % vector with values of speed in m/s
a = gradient(speed); % vector with values of acceleration in m/s2
max_speed = max(speed);
avg_speed = mean(speed); %average speed
tot_dist = sum(dist);
index = find(S>30);
sec_greater_than_30 = GPS_data(index,1);
tot_sec_greater_than_30 = sum(sec_greater_than_30);
disp(['Total number of seconds greater than 30 km/hr = ',num2str(tot_sec_greater_than_30)])
```

```
>> read_GPS_data
Total number of seconds greater than 30 km/hr = 111192
```

Problem 4

```
% Matix calculations
A = [2 3 5 7;
     2 9 7 5;
     6 4 2 1;
     3 1 2 6];
B = [10 20 18 14];
C = B*A;
D = B(1:3);
E = 45*B'+10;
F = A(:,3)+6;
G = diag(A);
H = diag(A)+B;
J = zeros(4,4)+2.5*A;
x = inv(A)*B';
```

a) Matrix B [1x4] is multiplied with matrix A [4x4]. The [1,4] element of matrix C is obtained as follows:
(10*2)+ (20*2) + (18*6) + (14*3) = 20 + 40 + 108 + 42 = 210

```
C =

   210   296   254   272
```

b) Matrix D retrieves the values in the first 3 columns of matrix B

```
D =

    10    20    18
```

c) B' is the transpose of matrix B which is given as

B' = [10
20
18
14]

The value of each row in the matrix is multiplied by 45 following which 10 is added to it.
The value of the first row in matrix E is obtained as follows:
(10*45) + 10 = 450 + 10 = 460

```
E =

   460
   910
   820
   640
```

d) A(:,3) gives the values of all the rows in the 3rd column. This is given as

A(:,3) = [5
7
2
2]

To the value of each row in this matrix, 6 is added.

The value of the first row in matrix F is obtained as follows:

5 + 6 = 11

```
F =

    11
    13
     8
     8
```

e) Matrix G is a column matrix of the diagonal values of matrix A

```
G =

     2
     9
     2
     6
```

f) In matrix H, the values in the each row is obtained by adding each column value of that row matrix B with the value in that row of matrix G.

The first two rows of matrix H can be obtained as follows:

H = [2+10 2+20 2+18 2+14
9+10 9+20 9+18 9+14 …..

```
H =

    12    22    20    16
    19    29    27    23
    12    22    20    16
    16    26    24    20
```

g) Zeros(4,4) gives a 4x4 matrix with all the values as zero. To this, 2.5 times the values in matrix A is added. So essentially, the final answer is the same as matrix A multiplied by 2.5.

The value of the first row in matrix J is obtained as follows:

0 + (2.5*20) = 0 + 5 = 5

```
J =

    5.0000    7.5000   12.5000   17.5000
    5.0000   22.5000   17.5000   12.5000
   15.0000   10.0000    5.0000    2.5000
    7.5000    2.5000    5.0000   15.0000
```

h) Matrix x is the multiplication of the inverse of matrix A and the transpose of matrix B
The inverse of matrix A is given as

```
inverse_matrix_A =

    0.1152   -0.1208    0.2022   -0.0674
   -0.5084    0.2893   -0.1124    0.3708
    0.7893   -0.2669    0.1910   -0.7303
   -0.2360    0.1011   -0.1461    0.3820
```

The transpose of Matrix B is given as

B' = [10
20
18
14]

The value of the first row in matrix x is obtained as follows:
= (10*0.1152) + (20*-0.1208) + (18*0.2022) + (14*-0.0674)
= 1.152 – 2.416 + 3.6396– 0.9436
= 1.432

```
x =

    1.4326
    3.8708
   -4.2303
    2.3820
```