

Assignment 8: Integration, Polynomials and Functions

Date Due: April 13, 2020

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Problem 1

Recycled High Performance Asphalt (RHPA) is an important material used in roadway construction. A civil engineer wants to estimate the likelihood that a given mixture of RHPA survives the repeated loads of a busy highway. Using laboratory samples, the engineer collects data and finds that the life expectancy of the RHPA can be approximated using a negative exponential distribution (see Figure 1). The equation of the Probability Density Function (PDF) of the negative exponential distribution is presented in statistics textbooks and repeated here for completeness (https://en.wikipedia.org/wiki/Exponential_distribution).

$$f(x) = \frac{1}{\beta} e^{-x/\beta}$$

where:

x = random variable - pavement life (years)

β = parameter of the distribution (mean pavement life)

$f(x)$ = probability that pavement survives x years

After testing 100 pavement samples, the engineer estimates the mean pavement life (i.e., parameter beta) is 11.3 years in the lab. The area under probability density function has a fundamental interpretation in random processes. For example, the area under the curve in Figure 1 can be interpreted as the probability that the RHPA pavement survives the repeated loading conditions when the area is calculated between 0 and a given pavement life time (a value in the x axis).

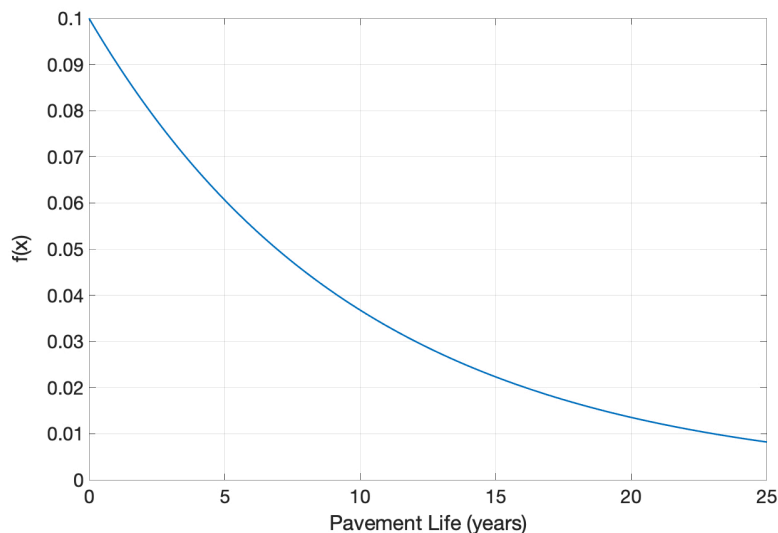


Figure 1. Sample Negative Exponential Density Function with Beta = 10 years. Interpretation of the Area Under the Curve.

Task 1

Create a Matlab script to evaluate the function $f(x)$ using the value of “beta” observed for the pavement samples. Plot the resulting PDF as a function of pavement life. Label accordingly. Verify that the PDF plot is consistent with the negative exponential distribution.

Task 2

Create a Matlab function to calculate the value of the negative exponential distribution PDF ($f(x)$). The function needs a single arguments: x (*the value of the random variable*). The output of the function is $f(x)$. Pass the value of beta to the function using a global variable as shown in class for the runoff example.

Task 3

Create another Matlab script to estimate the probably that a RHPA survives more than 10 years. This script uses the function created in Task 2 to evaluate the area under the curve.

Task 4

Modify the Matlab code to find the probability that the RHPA lasts pavement between 8 and 14 years.

Problem 2

You are in charge of a dredging operation to construct a waterway lock. Your task is to determine the amount of material to be removed in the dredging operation to create a 20 meter constant depth lock. The lock is expected to be 100 meters wide. The Table below shows the existing waterway depth (column 2) at stations spaced every 5 meters across the waterway (column 1).

Table 1. Waterway Stations and Depth.

Horizontal Coordinate (meters)	Vertical Profile (meters)
0	0.00
5	-2.90
10	-4.25
15	-5.10
20	-6.20
25	-7.15
30	-8.60
35	-9.90
40	-10.80
45	-11.30
50	-12.48
55	-12.10
60	-11.00
65	-10.20
70	-8.20
75	-7.30
80	-5.60
85	-3.85
90	-3.20
95	-2.10
100	0.00

Task 1

Create a Matlab script to read the data provided in the Table 1. Use any procedure learned in class to import the data into Matlab.

Task 2

Improve the script created in Task 1 to estimate the area under the cross section area of the existing waterway profile. Use the Newton-Cotes numerical integration algorithms learned in class.

Task 3

Modify the script created for Task 2 but using the Trapezoidal numerical integration algorithm. Comment on the differences observed.

Task 4

Further improve the script created in Task 3 to estimate the area needed for dredging. Remember, the lock is expected to have a 20 meter constant depth.

Problem 3

A civil engineer is designing an airport terminal (see Figure 2) using some basic aircraft characteristics that relate the aircraft wingspan and the aircraft weight.

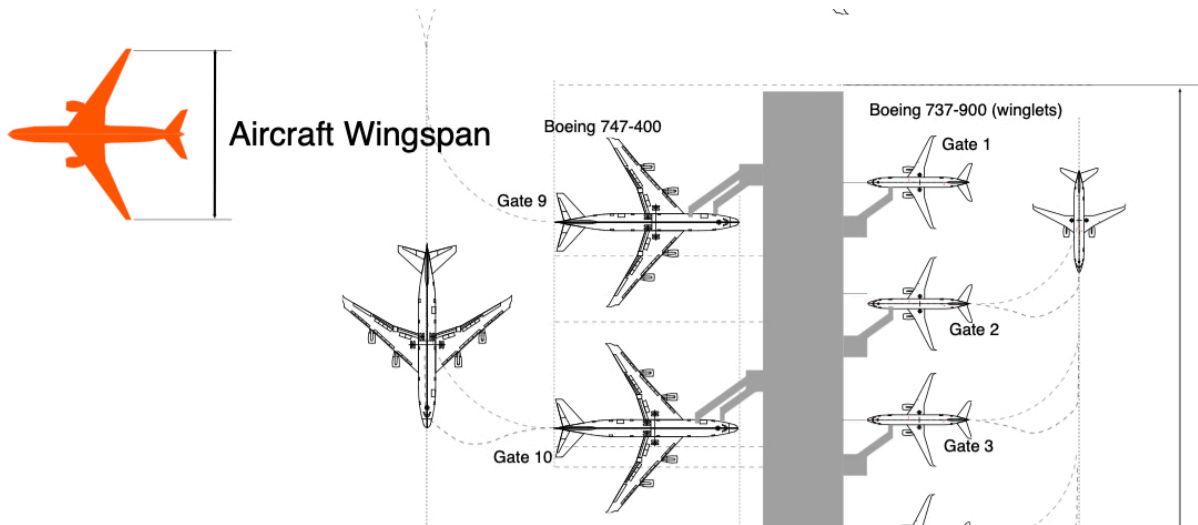


Figure 2. Partial View of a Simple Airport Terminal.

Use the aircraft data numeric file (see syllabus week 11) to solve the problem (the file is labeled: **Aircraft data numeric file**). A sample of the data file is shown in the following lines.

```
% Aircraft data file

% Contains the following information:
%
% Column 1 = takeoff weight (lb)
% Column 2 = approach speed (knots)
% Column 3 = wingspan (feet)
% Column 4 = tail height (ft)

7000 98 44.00 9.58
170000 123 156.25 48.50
1250 64 36.00 6.00
2050 67 37.50 10.33
```

Task 1

Create a Matlab script to read the numerical data and plot the aircraft wingspan (x-axis) against the aircraft takeoff weight (y-axis). Observe the relationship between the two variables. Use the interactive Basic Fitting functionality in Matlab, determine the best 2nd and 3rd order polynomials to approximate the data. In the solution, state the polynomials obtained.

Task 2

Create a Matlab script to estimate the coefficients of the best 2nd and 3rd polynomials (using least-square estimation) that relates the aircraft wingspan (x-axis) and the aircraft weight (y-axis). This analysis requires obtaining the polynomial coefficients using Matlab code. In your selection of the best polynomial

estimate the Sum of the Squared Errors (SSE) between the data points and the values using the polynomial. Decide which polynomial to use after trying various options (i.e., the solution with the minimum value of SSE is the one selected). Tabulate the values of SSE against the order of the polynomial used.

Problem 4

A simple equation to predict the so-called basic resistance of a high-speed rail system (R_{basic}) moving at speed (v) is given by Davis' equation and shown below. This equation includes the resistance due to aerodynamic drag and axle load friction resistance (i.e., resistance between train steel tires and the rail track).

$$R_{basic} = A + Bv + Cv^2$$

where: A , B , and C are train-specific coefficients and v is the train speed in meters per second. The values of A , B , and C are determined using wind tunnel testing and actual testing of the trains in test tracks. The values of A , B , and C for a Japanese High-Speed Train of the series Shinkansen 300 (see <http://en.wikipedia.org/wiki/Shinkansen>) are provided below with their corresponding units.

A = 7.91000; % units are kN
B = 0.10356; % units are kN s/m
C = 0.01099; % units are kN s-s/m-m

Note that the units of A , B and C are consistent to produce a basic resistance of the train in kiloNewtons.

The power required (P) to move the train at speed (v) is the product of Resistance (R) and speed (v).

Task 1

Create a Matlab function to estimate the basic resistance (R) (in Newtons) and the power required (P) (in Watts if R is estimated in Newtons) to move for the high-speed train for a given speed (v) in m/s. The function needs to produce two outputs (R and P).

Task 2

Create a Matlab script that calls the function created in Task 1. Using the script, estimate the train resistance and power required for speeds ranging from 20 to 80 m/s at intervals of 0.1 m/s. Make a plot of the resistance vs. speed and power vs speed using the subplot command.