Assignment 9: Simulink and Modeling Systems

Date Due: November 20, 2013 (Wednesday)   Instructor: Trani

Problem 1

The differential equation to predict the angular acceleration ($\ddot{\theta}$)of a rigid pendulum can be derived using a free body diagram and Newton's second law of motion shown in Figure 1.
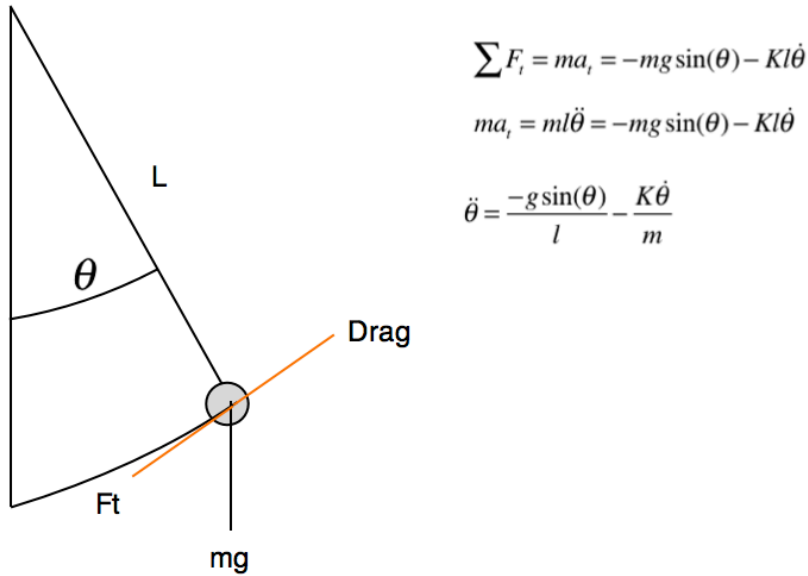

Figure 1. Free-Body Diagram for a Rigid Pendulum.

$$\sum F_t = ma_t = -mg\sin(\theta) - Kl\dot{\theta}$$

$$ma_t = ml\ddot{\theta} = -mg\sin(\theta) - Kl\dot{\theta}$$

$$\ddot{\theta} = \frac{-g\sin(\theta)}{l} - \frac{K\dot{\theta}}{m} \qquad\qquad (1)$$

where: K is the drag damping factor, θ is the angular position (in radians) between the vertical and the position of the rigid pendulum, $\dot{\theta}$ is the angular velocity, m is the mass of the pendulum (kg) and l is the length of the pendulum (meters). The units of the Equation (1) are consistent to make the units of $\ddot{\theta}$ rad/s2.

Task 1

Create a Simulink model that solves the set of two differential equations shown above. The objective is to calculate the angular position (θ) and the angular speed ($\dot{\theta}$) of the pendulum. Solve the differential equations for the following initial conditions:

$\dot{\theta} = 0$ and $\theta = 0.35$ radians. In this solution assume the values of K to be 0.40 kg/s, g=9.81 m/s², l = 0.80 meters, and m =53 kg.

Task 2

After running the Simulink model, export the angular position (θ) and the angular speed ($\dot{\theta}$) of the pendulum using a Simout block (in the Sinks library). Plot the values of the state variables position (θ) and the angular speed ($\dot{\theta}$) as a function of time (from 0-100 seconds). Label the plots accordingly. Also plot the angular acceleration and plot it with respect to time. Estimate the maximum velocity for the pendulum,for the given initial conditions.

Task 3
Make a plot of angular speed vs. angular position.

Task 4

Using the plots produced in Task C, estimate the period of oscillation of the pendulum. How long does it take for the pendulum to reach 5% of its initial angular position?

Problem 2

A civil engineer needs to model the dynamics of a building subject to seismic loads. The building and its foundation is modeled as series of spring and dampers as shown in Figure 2.
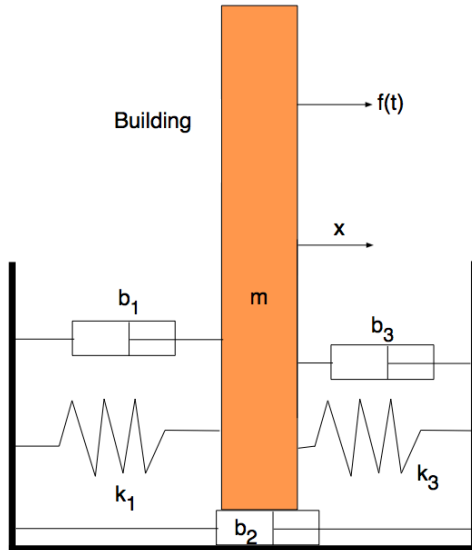

Figure 2. Building and Foundation System for Problem 2.

Task 1

The fundamental equation of motion for the acceleration of the building at the foundation is:

$$\ddot{x} = \frac{f(t)}{m} + \left(\frac{b_3 - b_2 - b_1}{m} \right)\dot{x} + \left(\frac{k_2 - k_1}{m} \right)x$$

where:

$\ddot{x} =$ lateral acceleration of the building system (m/s^2)

$\dot{x} =$ lateral speed of building system (m/s)

$x =$ lateral displacement of building system (m)

The values of the constants are:

m = 2000000;	% mass (kg)
b1 = 500000;	% damper constant (N / (m/s))
b2 = 550000;	% damper constant (N / (m/s))
b3 = 600000;	% damper constant (N / (m/s))
k1 = 9e6;	% spring constant (N/m)
k2 = 7e6;	% spring constant (N/s)

Task 2

Using Simulink, construct a model of the building system and solve the differential equations of motion of the building system to find the dynamic response of x and $\dot{x}$ as a function of time.

Task 3

Solve the dynamic response of the building system when is subject to an initial instantaneous displacement of 17 cm caused by a seismic jolt. In this analysis the forcing function $f(t)$ is zero as you explore the natural dynamics of the system. After running the Simulink model, export the lateral speed and lateral displacement of the building over time. Use a Simout block (in the Sinks library) to export the Simulink model results and to make the plots in Matlab. Plot speed and lateral displacement and record the maximum values observed. Find the time when the system lateral displacement has reached 5% of the original input displacement.

Task 4

Simulate a small seismic event by adding numerical values to the external forcing function $f(t)$ as shown in the table below. It is recommended that you model the time-varying external function using a table lookup function in Simulink. An example of a table lookup function is shown in the Simulink Examples (page 25) in the discussion of the runway arrestor system. Export the results from Simulink and plot speed and lateral displacement of the building system and record the maximum values observed.

Table 1. Forcing Function Values of Simulated Seismic Event.

Time of Event (seconds)	Force (N)
0.0	0.0
1.2	30000.0
4.6	420000.0
7.6	110000.0
10.2	450000.0
13.0	80000.0
18.0	360000.0
23.2	20000.0
25.3	30000.0
30.0	0.0
100.0	0.0

Problem 3

Work Problem 3 of Quiz 2 and solve the area under the terrain curve using a Newton-Cotes method (Quad function). IN this analysis, it is necessary that you approximate the terrain elevation data using a high-order polynomial. Then apply the Quad function.