Assignment 9: Dynamic Systems: Differential Equations

Date Due: May 1, 2020 at 5 PM Instructor: Trani

Problem 1

Consider the mass-spring-damper (MSD) system described in class and shown in Figure 1. The system has been demonstrated in class using the Matlab ODE solver. A 3000 kilogram object is attached to a 4000 N/m spring and a 2000 N/m/s damper as shown in Figure 1. This system is used to mitigate seismic loads in a small building.

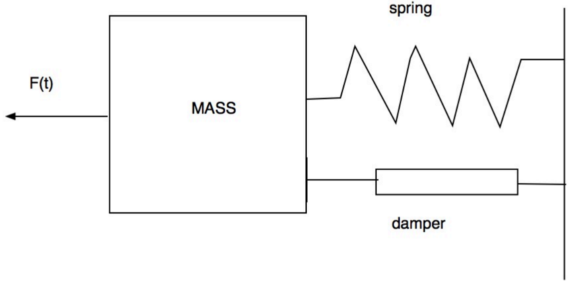

Figure 1. Mechanical System.

The equation of motion the system is presented below (2nd order differential equation):

$$m\ddot{x} = f(t) - kx - b\dot{x}$$

$$x = \text{system position (m)}$$

$$\dot{x} = \text{system velocity (m/s)}$$

$$\ddot{x} = \text{system acceleration (m/s-s)}$$

$$m = \text{mass (kg)}$$

$$k = \text{ Spring constant (N/m)}$$

$$b = \text{Damper constant (N/m/s)}$$

$$f(t) = \text{external force (N) - could be a function of time}$$

Task 1

Create a basic Simulink model to solve the differential equation(s) to calculate the position and velocity of the mass from a datum point. Solve the equations of motion if a 5000 Newton constant force - f(t) is applied to the MSD system simulating a shock applied to the MSD system after a small earthquake. Export the values of velocity, position and time from your Simulink model and make necessary plots to visualize the system. Use zero displacement and zero speed as initial conditions in your model.

Task 2

Solve the MSD using the Simulink 5th order method. Simulate until the MSD until the position of the mass is near its equilibrium condition (i.e., when oscillation peaks are very close to a steady-state condition). Create a Matlab script that takes the outputs of the Simulink model created in Task 1 and plot the position and velocity profiles of the MSD system as a function of time using two subplots in the same figure.

Using the plot, estimate how long will it take for the MSD system to reach a final displacement position within 5% of its long-term (equilibrium condition).

Task 3

Suppose that you are in charge of making design changes to the mechanical system. This type of system (called a seismic damper) is used in buildings to dampen oscillations due to earthquakes or wind (see article http://www.wind.arch.t-kougei.ac.jp/info_center/ITcontent/tamura/10.pdf). Your task is to specify the numerical value of a new damper (b) so that the MSD reaches 5% of its final displacement in less than 8 seconds. Note that since dampers are expensive, your task is to make the damper light, yet powerful to restore the system quickly to a steady-state condition. Assume a damper weighs 0.2 kg for each N/(m/s) value of the damper constant.

State the mass of the damper that satisfies the criterion.

Problem 2

Read the article Tuned Mass Dampers (https://en.wikipedia.org/wiki/Tuned_mass_damper) and answer the following questions.

a) Explain the principle of operation of tuned mass dampers in civil engineering.

b) Are there any examples of tuned mass dampers in tall structures? Name 3 examples.

c) Name the tallest structure in the World with a tuned mass damper.

d) Besides buildings, what other man-made structures employ tuned dampers? Provide an example.

Read the article: http://www.wind.arch.t-kougei.ac.jp/info_center/ITcontent/tamura/10.pdf and explain in a couple of paragraphs the meaning of design damping ratio. You can always reference other articles on the subject.

Problem 3

Figure 2 shows the runway arrestor system discussed in class.

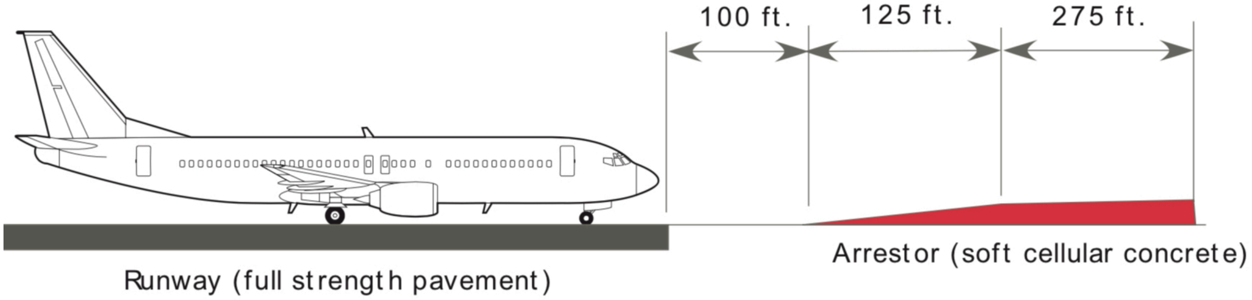

Figure 2. Runway Arrestor Bed for Problem 3.

Table 1 shows the laboratory test results of a new type of cellular concrete to be used in the an arrestor bed at Roanoke Regional Airport. The proposed dimensions of the arrestor bed are shown in Figure 2. The full depth of the arrestor bed is proposed to be 23 inches. The arrestor bed ramp increases the depth linearly from 0 to the full arrestor bed in 125 feet.

Task 1

Create a Matlab script to obtain a non-linear regression model (i.e., polynomial) to approximate the friction coefficient as a function of the arrestor bed depth. Clearly state the best polynomial fit to the data.

Task 2

Create a Simulink model to solve the differential equation(s) to calculate the position and velocity of the aircraft assuming an initial speed of 110 km/hr. Your Simulink model should use the polynomial regression created in Task 1. Export the values of velocity, position and acceleration vs. time from your Simulink model and make necessary plots to visualize the system. Estimate the distance traveled by the aircraft to come to a full stop with the given initial conditions.

Table 1. Coefficient of Friction Data Obtained in the Lab.

Depth of Arrestor Bed (inches)	Coefficient of Friction (Mu) (dim)
0.0	0.000
2.5	0.050
5.0	0.085
7.5	0.110
10.0	0.135
12.5	0.190
15.0	0.245
17.5	0.286
20.0	0.340
22.5	0.420
25.0	0.550
27.5	0.590

Task 3
Create a second Simulink model to solve the differential equation(s) to calculate the position and velocity of the aircraft assuming an initial speed of 110 km/hr. Your Simulink model should use the actual data of Table 1 using a table look-up function. Export the values of velocity, position and acceleration vs. time from your Simulink model and make necessary plots to visualize the system.

Task 4
Compare the solution obtained with models developed in Tasks 2 and 3. Comment if the proposed design with dimensions given in Figure 2 are acceptable (i.e., can the arrestor bed stop the aircraft at the design speed?).