Assignment 9: Ordinary Differential Equations, Integration and Polynomial Regression

Date Due: April 19, 2021

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Problem 1

Consider the train equation of motion presented on page 41h of the course notes Matlab Advanced Functions (http://128.173.204.63/courses/cee3804/cee3804_pub/Notes13_Matlab_inter_cee3804.pdf). The acceleration equation for the high-speed train is of the form:

$$\frac{dV}{dt} = C_1 + C_2 V + C_3 V^2$$

where:

$V =$ speed of the train (m/s)

C_1, C_2, C_3 are model constants

Units of model constants are consistent to produce $\frac{dV}{dt}$ in meters/second

The values of constants C_1, C_2, C_3 can be estimated from data collected by a railway engineer.

The acceleration data of the train is provided to you as two vectors (acceleration and speed data). Each pair of values presents the train acceleration (m/s^2) for a given speed (in m/s).

```
acceleration data = [2.2 1.4 1.1 0.89 0.75 0.51 0.38 0.26 0.13 0];   % in meters/second-second
speed data = [0 25 35 40 50 60 65 70 75 85];                           % in meters/second
```

Task 1

Create a Matlab script to estimate the values of coefficients C_1, C_2, and C_3. The script should include the polynomial regression analysis to determine the numerical values of the three coefficients. State the Goodness-of-Fit by looking at the value of R-square.

Task 2

Create a Simulink model to solve the differential equation of motion (shown above) and obtain a velocity profile for the train in the first 250 seconds after the train leaves the station. The initial conditions are: zero speed at the train station.

Export the values of velocity versus time from your Simulink model and make a plot of velocity vs. time in Matlab (Simulink plots are crude). Label your plot accordingly.

Task 3

Improve the model created on Task 2 to estimate the distance traveled by the train after leaving the station. Assume we want to measure the distance traveled from the train station (zero distance traveled at the train station). Export the values of distance from your Simulink model and make the plot of velocity vs. time and distance vs time in Matlab.

Task 4

Use the output of the Simulink model and the plots produced in Matlab to find the distance of the track needed to reach 70 m/s. Comment.

Problem 2

Read the article Tuned Mass Dampers (https://en.wikipedia.org/wiki/Tuned_mass_damper) and their application to buildings and related structures. Please answer the following questions.

a) Explain the principle of operation of tuned mass dampers and how they help civil engineers to build safer structures.

b) Are there any examples of tuned mass dampers in building structures? Name an example.

c) Besides buildings, what other man-made structures employ tuned dampers? Provide some examples.

Problem 3

Consider the mass-spring-damper (MSD) system described in class and shown in Figure 1. The system has been demonstrated in class using the Matlab ODE solver. A 4000 kilogram mass is attached to a 10000 N/m spring and a 2000 N/m/s damper as shown in Figure 1. This system is used to mitigate seismic loads in a small building.

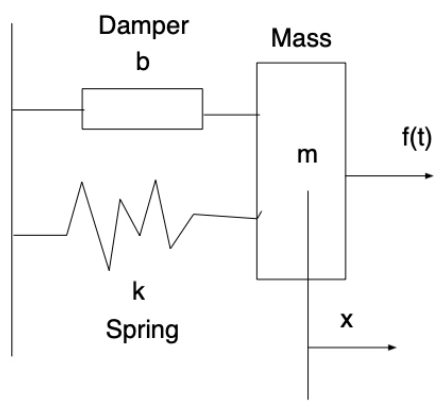

Figure 1. Mass-Spring-Damper System.

The equations that define the motion of the MSD system when disturbed by a force f(t) are:

$$\ddot{x} = \frac{1}{m}\left(f(t) - kx - b\dot{x}\right)$$

$x =$ system position (m)

$\dot{x} =$ system velocity (m/s)

$\ddot{x} =$ system acceleration $(m/s-s)$

$m =$ Mass (kg)

$k =$ Spring constant (N/m)

$b =$ Damper constant $(N/m/s)$

$f(t) =$ external force (N) $-$ could be a function of time

Task 1

Create a Simulink model to solve the differential equation(s) to calculate the position and velocity of the mass from a datum point. Solve the equations of motion if a constant 5000 Newton force - f(t) is applied to the MSD system simulating a shock applied to the MSD system after a small earthquake. Export the values of velocity, position and time from your Simulink model and make necessary plots (as a function of time) to visualize the system. Use zero displacement and zero speed as initial conditions in your model.

Task 2

Simulate until the MSD until the position of the mass is near its equilibrium condition (i.e., when oscillation peaks are very close to a steady-state condition). Export the Simulink results (speed and position of the MSD system) to Matlab. Create a Matlab script that takes the outputs of the Simulink model created in Task 1 and plot the position and velocity profiles of the MSD system as a function of time using two subplots in the same figure.

Using the plot, estimate how long will it take for the MSD system to reach a final displacement position within 5% of its long-term (equilibrium condition after a long period of time).

Task 3

Suppose that you are in charge of making design changes to the mechanical system. This type of system (called a seismic damper) is used in buildings to dampen oscillations due to earthquakes or aerodynamic loads (see article http://www.wind.arch.t-kougei.ac.jp/ info_center/ITcontent/tamura/10.pdf). Your task is to specify the numerical value of a new damper (b) so that the MSD reaches 5% of its final displacement in less than 5 seconds. Note that since dampers are expensive, your task is to make the damper light, yet powerful to restore the system quickly to a steady-state condition. Assume a damper weighs 0.2 kg for each N/(m/s) value of the damper constant.

State the mass of the damper that satisfies the criterion.

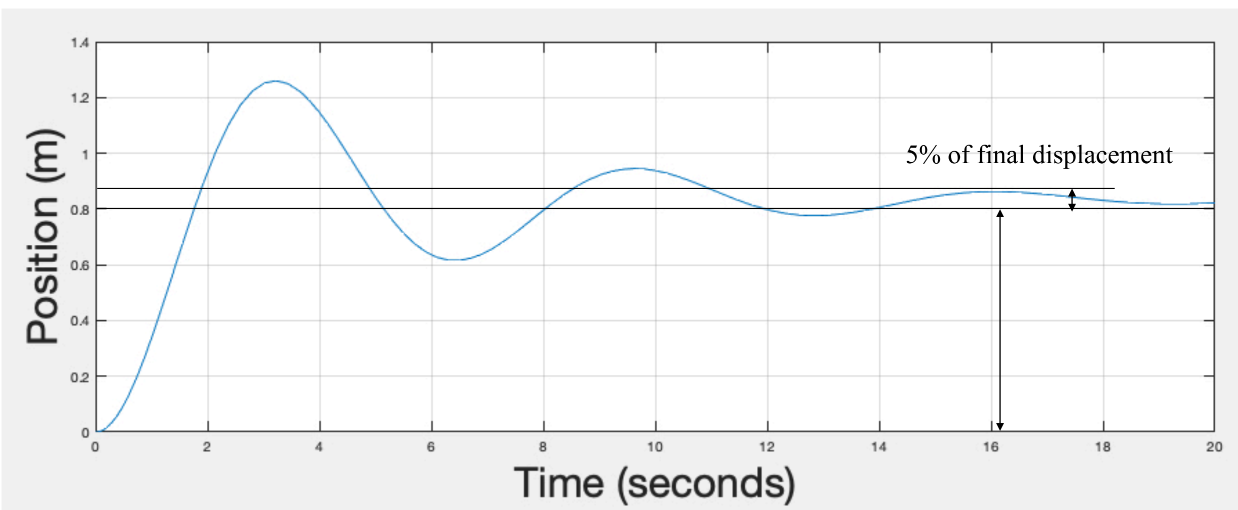

Figure 2. Position Response of MSD System after an External Force Introduces a Perturbation into the System.