Assignment 9: Integration and Differential Equations

Date Due: April 27, 2022   Instructor: Trani

Problem 1

Improvements to a waterway require estimation of the amount of material to be removed in the dredging operation. Table 1 shows the waterway depth (column 2) at stations spaced every 5 meters across the waterway (column 1). Figure 1 shows the desired dimension sow the waterway after dredging. A minimum depth of 15 meters is required to allow larger contained ships to transit.

Table 1. Waterway Stations and Depth.

Horizontal Coordinate (meters)	Vertical Profile (meters)
0	0.00
5	-2.40
10	-4.10
15	-5.00
20	-6.20
25	-7.15
30	-8.60
35	-9.90
40	-10.10
45	-10.60
50	-10.90
55	-10.50
60	-10.90
65	-10.20
70	-8.50
75	-7.30
80	-5.60
85	-4.90
90	-3.40
95	-2.10

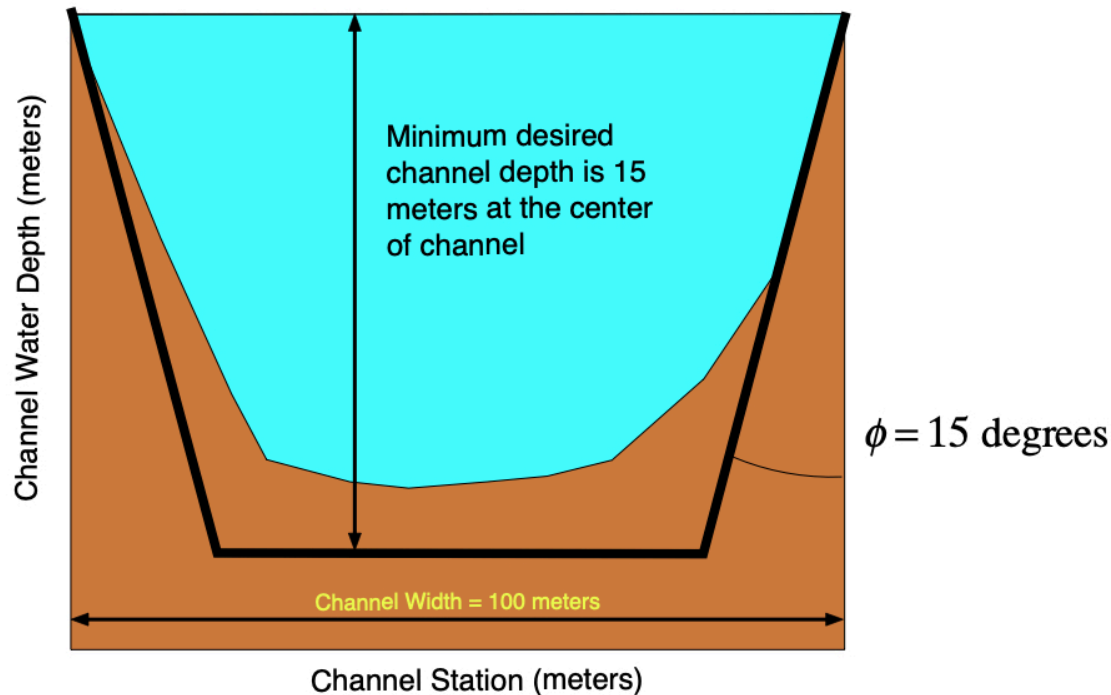| Horizontal Coordinate (meters)	Vertical Profile (meters)
100	0.00

Figure 2. Waterway Cross Section and Desired Dimensions.

Task 1
Create an Excel file with the station and water depth data of the existing channel. Create Matlab script to read the data provided in the Table 1. Use any importing procedure learned in class.

Task 2
Improve the script created in Task 1 to estimate the area under the cross section area of the existing waterway profile. Use the trapezoidal rule in your estimation.

Task 3
Further improve the script in Task 3 to estimate the area needed for dredging. Your code should be able to estimate the coordinates of the new profile (with a flat bottom) as a function of two parameters: channel depth and slope angle (phi). Run the code with channel depth of 15 meters and phi=15 degrees. Display the answer in the command window with appropriate labels and units.

Task 4
Improve the script created in Task 3 and plot the existing channel profile and the new profile after dredging. improve the script in Task 3 to estimate the area needed for dredging.

Task 5
Run the script created in Task 3 with channel depth of 20 meters and phi= 17 degrees.

Problem 2

Asphalt is a very important material to civil engineers. The fabrication of asphalt requires the material to be heated to allow better mixing. The first order differential equation (dT/dt) that estimates the rate of change of temperature of concrete as a function of time is:

$$\frac{dT}{dt} = -K(T - T_a)^m$$

$T =$ asphalt mix temperature (deg. C)

$K =$ heat release constant (1/minute)

$T =$ temperature of asphalt material (deg. C)

$T_a =$ ambient temperature (deg. C)

$m =$ constant of proportionality (dim)

Task 1
Create a Simulink model (block diagram) to solve the problem.

Task 2
Test the Simulink model created in Task 1 using using values for K for asphalt is 0.043 (1/minute), m = 1.8 and ambient temperature is 18 deg. C.Assume the initial temperature of the asphalt is 205 degrees.

Task 3
Plot the results of Task 2 in Matlab by exporting the answers to the Matlab workspace and making a plot of asphalt temperature versus time.

Task 4
Compare the numerical solutions of the Simulink model using the Runge-Kutta 3rd order and the Runge-Kutta 5th oder solutions.

Problem 3

Plywood is an important material used in construction. A civil engineer wants to estimate the distribution of the Modulus of Elasticity (MOE) for various specimens of Hickory beams received from a factory. The engineer collects data and finds that the modulus of elasticity is normally distributed (see picture below). The equation of the Probability Density Function (PDF) of the Gaussian (or Normal) distribution is given in any statistics textbook and repeated here for completeness (http://en.wikipedia.org/wiki/Normal_distribution).

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left[\frac{(x-\mu)^2}{2\sigma^2}\right]}$$

where: x is the random variable in question (modulus of elasticity in psi) and μ and σ are the mean and standard deviation (in psi) of the random phenomena modeled. After testing 1,000 samples, the engineer estimates the mean modulus of elasticity to be 1.93 e6 psi. The standard deviation is found to be 6.4e4 psi.

The area under probability density function has a fundamental interpretation in random processes. For example, the area under the curve in Figure 2 can be interpreted as the probability that random variable x will take values between **a** and infinity.
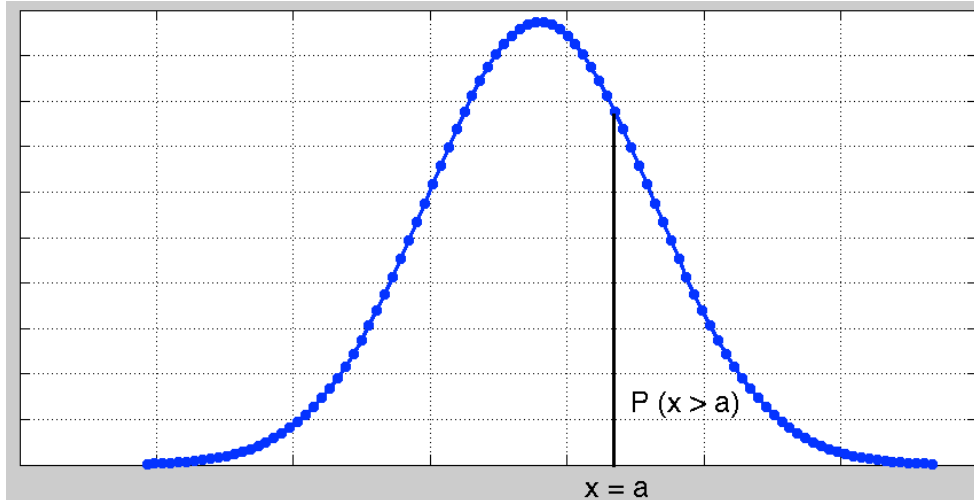
Figure 2. Sample Normal Probability Density Function. Interpretation of the Area Under the Curve.

Task 1

Create a Matlab script to evaluate the function f(x) using the values of μ and σ observed for Hickory beams. Plot the resulting PDF as a function of modulus of elasticity. Label accordingly. Verify that the PDF plot is the well-known bell-shape curve of the Normal distribution.

Task 2

Create a Matlab function to calculate the value of the Normal distribution PDF (f(x)). The function needs two take three arguments: *x (the value of the random variable)*, μ and σ. The output of the function is *f(x)*.

Task 3

Create another Matlab script to estimate the probably that wood samples MOE are greater than 2.00e6 psi, the minimum design specification.

Task 4

Find the probability that a lot of 1,000 of random sample beams have MOE values ranging from 2.00e6 psi to 2.20e6 psi.