

Quiz 1 Solution

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Write your solutions in a single Word document and convert to PDF file. Cut and Paste all your answers using screen captures. Show all your work. Label your file with your last name and CEE3804. Email the solution to vuela@vt.edu and to Carol (yqliang@vt.edu).

Problem 1 (25 Points)

An engineer formulates a linear programming problem as follows:

Maximize

$$Z = 40x_1 + 70x_2$$

subject to:

$$x_2 \leq 135$$

$$x_1 + 1.8x_2 \leq 300$$

and the non-negativity constraints $x_1 \geq 0$ and $x_2 \geq 0$

- a) Formulate the problem in standard Linear Programming form to solve the problem using the Simplex Method (add slack and artificial variables as needed)

$$\text{Max } Z = 40x_1 + 70x_2$$

Constraint equations with additional slack variables

$$x_2 + x_3 = 135$$

$$x_1 + 1.8x_2 + x_4 = 300$$

$$x_1, x_2 \geq 0$$

- b) Find the first two **tables** of the solution using the Simplex Method. This includes the first table with the initial solution and the second table with one iteration.

Construct the first table.

| BV | Z | x1 | x2 | x3 | x4 | RHS |
|----|---|----|-----|-----|----|-----|
| Z | | 1 | -40 | -70 | 0 | 0 |
| x3 | | 0 | 0 | 1 | 1 | 135 |
| x4 | | 0 | 1 | 1.8 | 0 | 300 |

Step 1. Decide on the entering variable to the solution. x_2 enters the solution first (highest negative coefficient in Z-row).

Step 2. Apply ratio test to decide which Basic Variable leaves the solution first. Variable x_3 has the lowest ratio. Therefore x_3 leaves the solution and x_2 enters the solution.

| BV | Z | x_1 | x_2 | x_3 | x_4 | RHS | Ratio |
|-------|---|-------|-------|-------|-------|-----|--------|
| Z | 1 | -40 | -70 | 0 | 0 | 0 | |
| x_3 | 0 | 0 | 1 | 1 | 0 | 135 | 135 |
| x_4 | 0 | 1 | 1.8 | 0 | 1 | 300 | 166.67 |

Step 3. Perform row operations using the pivot row to eliminate all the coefficients of the pivot column. The table below shows the first solution to the problem. In this solution, variable x_2 has replaced x_3 . The current solution has a value of $Z=9,450$ with variables $x_1=0$, $x_3=0$, $x_2=135$, and $x_4=57$.

| BV | Z | x_1 | x_2 | x_3 | x_4 | RHS |
|-------|---|-------|-------|-------|-------|------|
| Z | 1 | -40 | 0 | 70 | 0 | 9450 |
| x_2 | 0 | 0 | 1 | 1 | 0 | 135 |
| x_4 | 0 | 1 | 0 | -1.80 | 1 | 57 |

Step 4. Repeat the procedure and check for optimality. Note the coefficient of x_1 is negative, therefore the solution can be improved by bringing x_1 to the solution.

| BV | Z | x_1 | x_2 | x_3 | x_4 | RHS | Ratio |
|-------|---|-------|-------|-------|-------|------|-------|
| Z | 1 | -40 | 0 | 70 | 0 | 9450 | |
| x_2 | 0 | 0 | 1 | 1 | 0 | 135 | N/A |
| x_4 | 0 | 1 | 0 | -1.80 | 1 | 57 | 57 |

Variable x_1 enters the solution and variable x_4 leaves the solution (Basic Variable set).

| BV | Z | x_1 | x_2 | x_3 | x_4 | RHS |
|-------|---|-------|-------|-------|-------|-------|
| Z | 1 | 0 | 0 | -2 | 40 | 11730 |
| x_2 | 0 | 0 | 1 | 1 | 0 | 135 |
| x_1 | 0 | 1 | 0 | -1.80 | 1 | 57 |

In this solution the value of the objective function row (Z-row) 11,730. The values of the decision variables are: $x_1=57$, $x_2 = 135$, $x_3 = 0$, $x_4 = 0$. the solution is not optimal because it contains a negative coefficient for variable x_3 in the objective function (Z-row).

| BV | Z | x1 | x2 | x3 | x4 | RHS | Ratio |
|----|---|----|----|-------|----|-------|-------|
| Z | 1 | 0 | 0 | -2 | 40 | 11730 | |
| x2 | 0 | 0 | 1 | 1 | 0 | 135 | 135 |
| x1 | 0 | 1 | 0 | -1.80 | 1 | 57 | N/A |

The previous table shows that variable x3 enters the solution and x2 leaves the solution. This solution is optimal with the value of Z at 12,000. The values of the decision variables are: x1=300, x2 = 0, x3 = 135, x4 = 0.

| BV | Z | x1 | x2 | x3 | x4 | RHS |
|----|---|----|------|------|----|-------|
| Z | 1 | 0 | 2 | 0 | 40 | 12000 |
| x3 | 0 | 0 | 1 | 1 | 0 | 135 |
| x1 | 0 | 1 | 1.80 | 0.00 | 1 | 300 |

Problem 2 (25 Points)

- c) Use **Excel Solver** to obtain the optimal solution of Problem 1. State the optimal value of Z and the values of x_1 and x_2 .

Show screen captures of your Excel Solver solution.

| Optimization Problem | | | |
|-----------------------------|--|----------------|-----|
| Decision Variables | | | |
| x1 | | 300 | |
| x2 | | 0 | |
| Objective Function | | | |
| 40 x1 + 70 x2 | | 12000 | |
| Constraint Equations | | | |
| | | Formula | |
| x2 <= 135 | | 0 <= | 135 |
| x1+1.8 x2 <= 300 | | 300 <= | 300 |
| x1 >= 0 | | 300 >= | 0 |
| x2 >= 0 | | 0 >= | 0 |

Solver Parameters

Set Objective:

To: Max Min Value Of:

By Changing Variable Cells:

Subject to the Constraints:

- \$B\$14 <= \$D\$14
- \$B\$15 <= \$D\$15
- \$B\$16 >= \$D\$16
- \$B\$17 >= \$D\$17

Make Unconstrained Variables Non-Negative

Select a Solving Method:

Problem 3 (25 Points)

A file named cruiseLine_Data.xls contains information about passenger cruise ships that operated at US ports between 2004 and 2012. A sample of the data is presented below.

| North American Cruises, January 1, 2004 - March 31, 2012 | | | | | | |
|--|-------------------------|-----------|-----------------|--------------------|--------|------------|
| Cruise Line | Vessel Name | Sail Date | Departure Port | Destination | Nights | Passengers |
| Carnival Cruise Line | CARNIVAL IMAGINATION | 1/1/04 | Miami | Western Caribbean | 4 | 2,449 |
| Royal Caribbean International | ENCHANTMENT OF THE SEAS | 1/1/04 | Fort Lauderdale | Western Caribbean | 4 | 2,262 |
| Carnival Cruise Line | CARNIVAL FANTASY | 1/1/04 | Port Canaveral | Bahamas | 3 | 2,636 |
| Carnival Cruise Line | CARNIVAL SENSATION | 1/1/04 | Tampa | Western Caribbean | 4 | 2,483 |
| Celebrity Cruise Line | CELEBRITY GALAXY | 1/2/04 | San Juan | Southern Caribbean | 7 | 1,644 |
| Royal Caribbean International | MAJESTY OF THE SEAS | 1/2/04 | Miami | Bahamas | 3 | 2,570 |
| Carnival Cruise Line | CARNIVAL SPIRIT | 1/2/04 | Miami | Southern Caribbean | 8 | 2,324 |
| Carnival Cruise Line | CARNIVAL ECSTASY | 1/2/04 | Long Beach | Mexico (Pacific) | 3 | 2,474 |
| Carnival Cruise Line | CARNIVAL FASCINATION | 1/2/04 | Miami | Bahamas | 3 | 2,484 |
| Princess Cruises | CORAL PRINCESS | 1/2/04 | Fort Lauderdale | Western Caribbean | 10 | 1,982 |

- a) Use a Pivot table to summarize the average number of nights per trip by cruise line. Which cruise line has the highest average nights per trip?

| Row Labels | Average of Nights |
|-------------------------------|-------------------|
| AIDA Cruise Line | 14.00 |
| Aida Cruises | 9.36 |
| Azamara Club Cruises | 11.00 |
| Carnival Cruise Line | 5.44 |
| Celebrity Cruise Line | 8.42 |
| Costa Cruise Line | 8.16 |
| Crystal Cruises | 11.59 |
| Cunard Line | 8.87 |
| Disney Cruise Line | 4.79 |
| Fred Olsen Cruise Line | 14.64 |
| Holland America Line | 8.98 |
| MSC Italian Cruises | 9.42 |
| Norwegian Cruise Line | 7.04 |
| Oceania Cruises | 13.39 |
| Princess Cruises | 8.79 |
| Regent Seven Seas Cruises | 8.78 |
| Royal Caribbean International | 5.97 |
| Seabourn Cruise Line | 11.24 |
| SeaDream Yacht Club | 7.17 |
| Silversea Cruises | 9.82 |
| Windstar Cruises | 7.58 |
| Grand Total | 6.83 |

The cruise line with longer cruise is Fred Olsen Cruise Line with 14.64 nights per cruise.

- b) Use a pivot chart to summarize the average number of passengers per cruise trip (i.e., passenger column) and by cruise line.

| Row Labels | Average of Passengers |
|-------------------------------|-----------------------|
| AIDA Cruise Line | 1,272.17 |
| Aida Cruises | 1,546.00 |
| Azamara Club Cruises | 640.23 |
| Carnival Cruise Line | 2,617.33 |
| Celebrity Cruise Line | 2,030.97 |
| Costa Cruise Line | 2,313.04 |
| Crystal Cruises | 694.12 |
| Cunard Line | 2,357.30 |
| Disney Cruise Line | 2,701.33 |
| Fred Olsen Cruise Line | 1,045.55 |
| Holland America Line | 1,558.45 |
| MSC Italian Cruises | 1,973.00 |
| Norwegian Cruise Line | 2,231.09 |
| Oceania Cruises | 700.44 |
| Princess Cruises | 2,484.78 |
| Regent Seven Seas Cruises | 496.02 |
| Royal Caribbean International | 2,788.94 |
| Seabourn Cruise Line | 200.89 |
| SeaDream Yacht Club | 89.94 |
| Silversea Cruises | 309.74 |
| Windstar Cruises | 150.55 |
| Grand Total | 2,339.18 |

Problem 4 (25 Points)

Steel cables are key components of suspension bridges to support the horizontal roadway/transit loads. For simplicity, we assume a uniformly distributed load along the horizontal span of the bridge. The tension at mid-span of the suspension bridge is given by the formula:

$$H = \frac{wL^2}{8d}$$

where:

H = Mid-span tension (Newtons)

w = Load per horizontal distance (N/m)

L = bridge span (m)

d = sag of suspension bridge cable (m)

Write a simple **function** in Excel to calculate the value of H given values of w , L and d (arguments of the function). Test the function and state the answer using the following values:

$w = 30,000$ N/m.

L = 100 meters

D = 20 meters

Show a screen capture of your Excel function in VBA.

| | A | B | C | D |
|----|--|-----------|---|---|
| 1 | Problem 4 | | | |
| 2 | Quiz 1 | | | |
| 3 | | | | |
| 4 | H = Mid-span tension (N) | | | |
| 5 | w = load per horizontal distance (N/m) | | | |
| 6 | L = bridge span (m) | | | |
| 7 | d = sag of the suspension bridge (m) | | | |
| 8 | | | | |
| 9 | w | 30000 N/m | | |
| 10 | L | 100 m | | |
| 11 | d | 20 m | | |
| 12 | H | 1875000 N | | |

```
Public Function tension(w, L, d)
' H = Mid-span tension (N)
' Input parameters to estimate mid-span tension
' w = load per horizontal distance (N/m)
' L = bridge span (m)
' d = sag of the suspension bridge (m)

tension = w * L ^ 2 / (8 * d)

End Function
```

VBA Code