Quiz 2 (55 minutes)

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Open book and notes, use of computer is allowed

Your Name : Solution

Problem 1 (30 Points)

You are a civil engineer and your task is helping in the reconstruction a high-speed train accident. Fortunately, no major injuries were reported to passengers. The data from the train data recorder is provided to your team. The data is presented in a separate file called "accidentTrace.m". the data has the following information:

% File with data recorder of a train accident

% Column 1 = time (seconds)
% Column 2 = acceleration (meters/second-second)

```
     0   -2.4720
  0.2000  -2.2651
```

Task 1

Create a Matlab script to read the accident data. In your script create two variables to keep the time and acceleration of the train data separately. Plot the accident data (time in the x axis and acceleration in the y axis).

```
1      % Script to analyze high-speed train
2      % Load the data file
3
4 -    load accidentTrace.m
5
6      % File with flight data recorder of a train accident
7      % Column 1 = time (seconds)
8      % Column 2 = acceleration (meters/second-second)
9      %      0  -2.4720
10     %   0.2000  -2.2651
11
12     % Rename the variables
13 -   time        = accidentTrace(:,1);
14 -   acceleration = accidentTrace(:,2);
15
16     % Make a plot
17 -   plot(time,acceleration,'or')
18 -   xlabel('Time (s)','fontsize',20)
19 -   ylabel('Acceleration (m/s-s)','fontsize',20)
20 -   grid
```
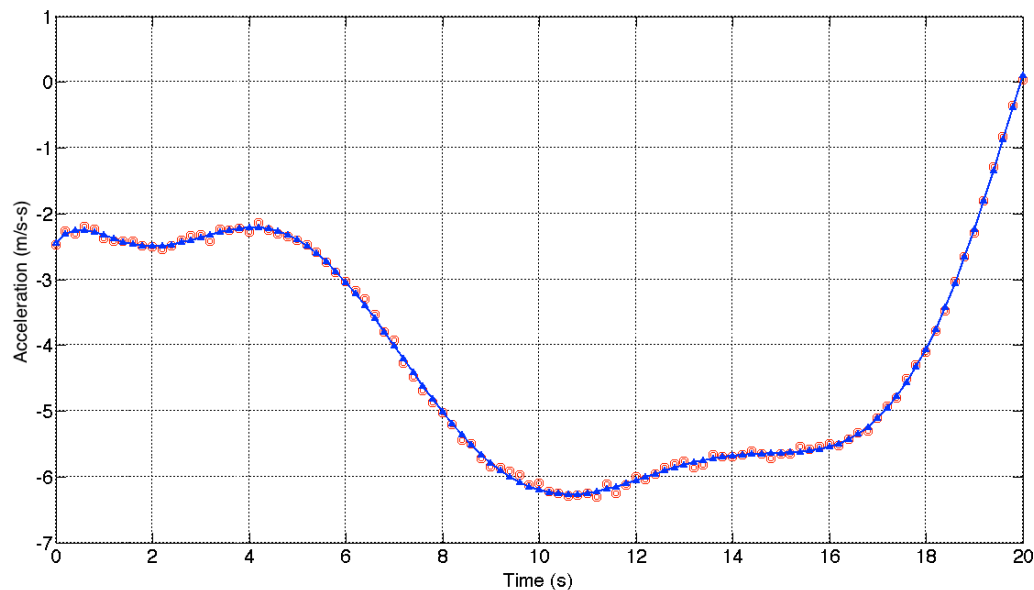

Figure 1. 9th Order Polynomial Fit to the Data.

Task 2

Using Matlab, obtain the best polynomial fit to approximate the acceleration-time data. Tell me how do you selected the order of the polynomial. Plot the polynomial approximation and compare it (in the same plot) with the answer obtained in Task 1.

```
21
22          % Make a polynomial regression – I checked visually using the Basic Fitting
23          % in the plot first. The norm for 9th and 10th order polynomials are very
24          % close
25          coeff = polyfit(time,acceleration,9);     % 9th order polynomial
26
27          % Evaluate the polynomial at points of time given and find residuals
28          accelApproximation = polyval(coeff,time);
29          residual = acceleration – accelApproximation;     % difference between data and regressi
30
31          % Replot the polynomial approximation using a blue line and triangles as
32          % the markers
33          hold on
34          plot(time,accelApproximation,'^-b')
```

Two methods to decide on the order of the polynomial: a) Calculate the sum of square errors (or the square of the residuals) from the data or b) estimate the norm of the residuals. The residuals or the vertical distance between the fitted data and the regression equation are shown graphically in the Matlab Basic Fitting panel (see Figure 3). They can be calculated as the difference between each data point of acceleration and the values of acceleration approximated by the regression equation. I calculated the residuals in line 29 above.

$$r_i = a_i - ap_i$$

r_i = residual at point i

a_i = acceleration at point i from data

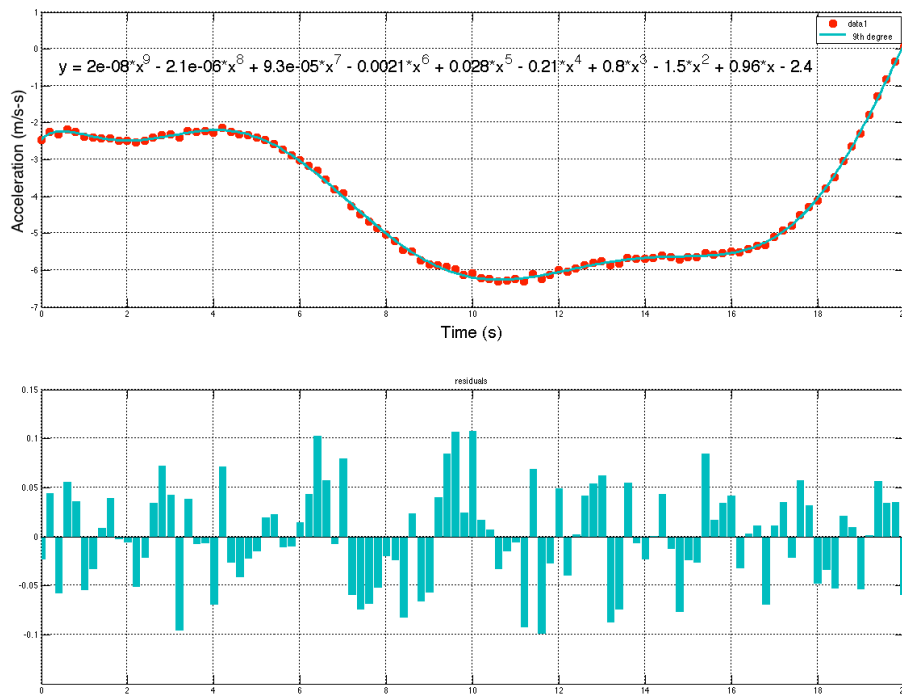ap_i = acceleration at point i estimated by regression curve

Figure 2. 9th Order Polynomial Fit to the Data and the Residuals (differences between data and the regression curve).
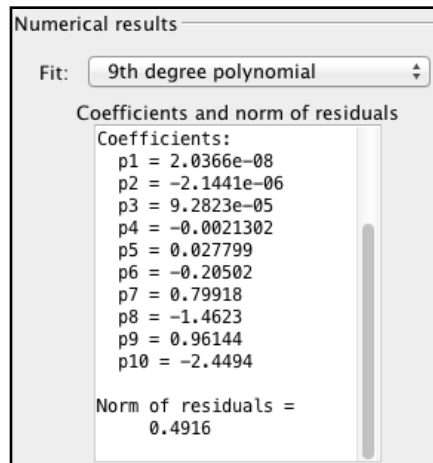
Numerical results

Fit: 9th degree polynomial

Coefficients and norm of residuals

```
Coefficients:
  p1 = 2.0366e-08
  p2 = -2.1441e-06
  p3 = 9.2823e-05
  p4 = -0.0021302
  p5 = 0.027799
  p6 = -0.20502
  p7 = 0.79918
  p8 = -1.4623
  p9 = 0.96144
  p10 = -2.4494

Norm of residuals =
      0.4916
```

Figure 3. Coefficients of 9th Order Polynomial Fit to the Data and the Value of the Norm.

The norm of the residuals is also provided directly in the Basic Fitting panel (see Figure 3).

Problem 2 (30 Points)

A formula used in Geotechnical Engineering to estimate the void ratio - a measure of the compaction level of soil - is given in Equation (1),

$$e = \frac{G_s \gamma_w (1 + w/100)}{\gamma} - 1 \tag{1}$$

where:

e = void ratio (dimensionless)

G_s = Absolute specific gravity of soild material (dimensionless)

γ = soil material unit weight (kN/m^3)

γ_w = water specific gravity (9.81 kN/m^3)

w = is the water (or moisture) content of the soil (%)

Task 1

Write a **function** in Matlab to calculate the value of e given the values of all four variables in the right hand side of Equation (1). The Matlab function should produce **two output variables**: 1) the value of e and 2) a Binary variable (called "binaryStd") that indicates if the value of e meets compaction standards. If the value of e is less than 0.8, the variable "binaryStd" should be set to 1. Otherwise it should be zero.

```matlab
function [ e, binaryStd ] = voidRatio(w,gamma,gammaW,Gs )
% Function to calculate the void ratio (e)

    e = Gs * gammaW * (1+w/100) / gamma -1;    % void ratio

    if e < 0.8
        binaryStd = 1;
    else
        binaryStd = 0;
    end

end
```

Task 2

Write a Matlab script to test the function created in Task 1. Test your function using the following values: $\gamma = 18.9 \text{ kN/m}^3$, $w = 10.6\%$, $\gamma_w = 9.81 \text{ kN/m}^3$ and $G_s = 2.46$.

```matlab
% Script to test the function void ratio
% Define values to calculate void ratio

w        = 10.6; % percent
gamma    = 18.9; % kN/m-m-m
gammaW   = 9.81; % kN/m-m-m
Gs       = 2.46  % dim

% Calculate void ratio
[ e, binaryStd ] = voidRatio(w,gamma,gammaW,Gs );

% Display the void ratio
disp(['Void ratio = ', num2str(e) , '   dimensionless' ])
```

Void ratio = 0.4122   dimensionless

Problem 3 (40 Points)

A civil engineer wants to calculate the amount of earth material to be removed for a highway project. The file "terrain2.m" contains the station and elevations of a cross section of the terrain in the area where the road will be constructed. The file is as follows:

% File that contains a 2-D set depicting the terrain features in a section of a highway
% construction project

% Column 1 = station coordinate (feet)
% Column 2 = elevation of terrain (feet)

```
  0   99.9010
  1.0000  101.1619
  2.0000  101.5920
  3.0000  102.2384
  4.0000  103.8551
```

Task 1

Create a Matlab script to read the terrain data. In your script create two variables to keep the station and elevation data separately. Plot the terrain data (station in the x axis and elevation in the y axis).

```matlab
% Script to analyze high-speed train
% Load the data file

load terrain2.m

% File that contains a 2-D set depicting the terrain features in a section of a highway
% construction project
% Column 1 = station coordinate (feet)
% Column 2 = elevation of terrain (feet)
%    0   99.9010
%    1.0000  101.1619

% Rename the variables
station      = terrain2(:,1);
elevation    = terrain2(:,2);

% Make a plot
plot(station,elevation,'o-k')
xlabel('Station (ft)','fontsize',20)
ylabel('Elevation (ft)','fontsize',20)
grid
```
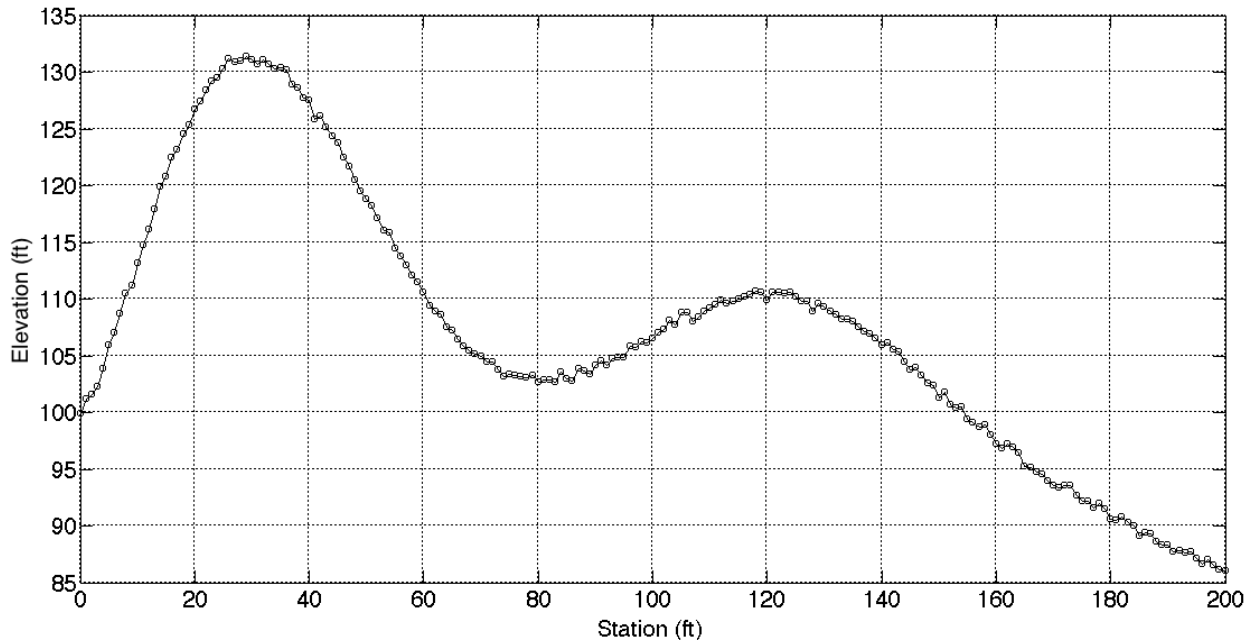
Figure 4. Complete Terrain Data Plot.

Task 2

Use the simplest Matlab numerical integration function to estimate the "area under the terrain function" for this cross section of the road project. It is known that in this section of the road, the desired elevation of the road is expected to be 100 feet above sea level. Terrain elevations below 100 feet will not be considered in the analysis.

```matlab
% Find the indices of the elevations above 100 feet
indicesAbove100 = find(elevation>=100);

% Get the statations and elevation data whose elevations are above 100 feet
elevationsAbove100 = elevation(indicesAbove100);
stationsAbove100   = station(indicesAbove100);
minStation = min(stationsAbove100);              % find the minimum station whose elevation
maxStation = max(stationsAbove100);              % find the maximum station whose elevatio

% Make a plot of stations and elevation
figure
plot(stationsAbove100,elevationsAbove100,'o-b')
xlabel('Station (ft)','fontsize',20)
ylabel('Elevation (ft)','fontsize',20)
grid

% Use the Matlab Trapezoidal rule to integrate the function
areaUnderTerrainCurve = trapz(stationsAbove100,elevationsAbove100) - ...
   (maxStation - minStation)*100;

clc
disp(['Area Under the Terrain Curve Above 100 feet is = ',num2str(areaUnderTerrainCurve) ])
```
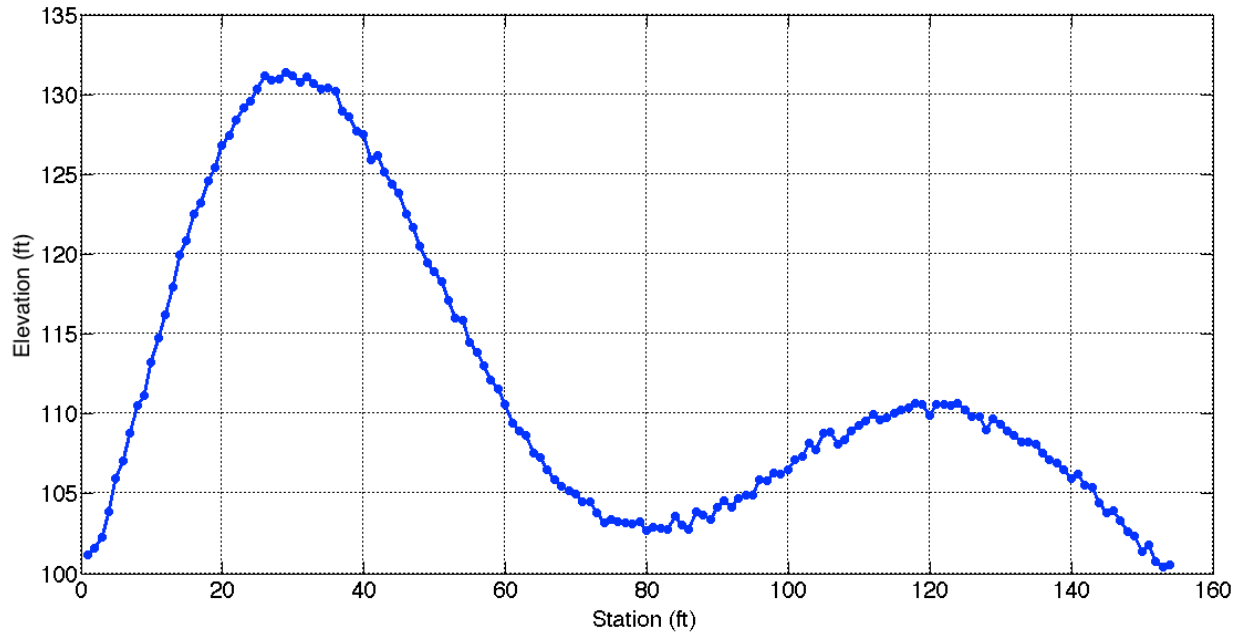
Figure 5. Terrain Data with Elevations Above 100 feet.

Area Under the Terrain Curve Above 100 feet is = 1835.7517 ft²