CEE 5614: Analysis of Air Transportation Systems   Fall 2015

Quiz # 2

Solution   Instructor: Trani

Problem 1

A regional airport has a four-corner post system to handle arrivals and departures as shown in Figure 1.
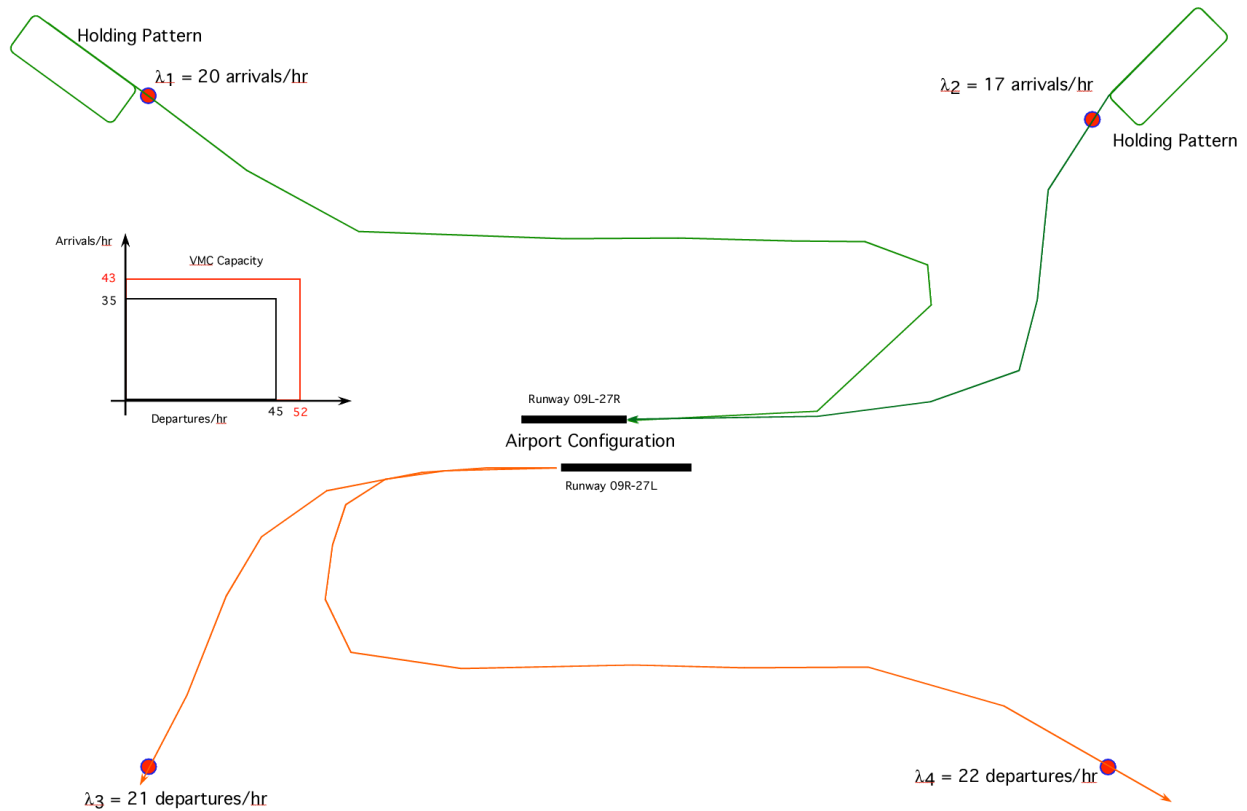

Figure 1. Four Corner Post System for Airport in Problem 1. The holding patterns in the North fixes of the airport are flown at FL 200 (NE quadrant) and FL 240 (NW quadrant). Orange Pareto Diagram Presents the VMC Capacity. Black Pareto Diagram Presents the IMC Capacity.

   a) If the arrivals and departures shown in the Figure occur randomly (Poisson) calculate the **delays experienced** in the typical day for both IMC and VMC conditions. Assume the service times that drive the capacity are negative exponential. Runway capacities are shown in Figure 1. Assume there is a constant stream of aircraft flying into this airport with values of lambda shown in Figure 1.

Recognize that random means there is a Poisson distribution of arrivals and departures. This calls for a queueing model with stochastic variables. The runways are used in segregated mode so easy runway can be studied as a single server.

For arrivals:

$$\lambda_{arr} = \lambda_1 + \lambda_2$$
$$\lambda_{arr} = 37 \text{ arrivals per hour}$$

The service rates (μ) are 35 for IMC conditions and 43 for VMC conditions. Using the stochastic queueing equations.

$\rho = \lambda / \mu$ Utilization factor

$$P_0 = \frac{1}{1 + \sum_{n=1}^{\infty} \rho^n} = \left(\sum_{n=0}^{\infty} \rho^n \right)^{-1} = \left(\frac{1}{1-\rho} \right)^{-1} = 1 - \rho$$

$$P_n = \rho^n P_0 = (1-\rho)\rho^n \quad \text{for n} = 0,1,2,3,.....$$

$L = \dfrac{\lambda}{\mu - \lambda}$ expected number of entities in the system

$L_q = \dfrac{\lambda^2}{(\mu - \lambda)\mu}$ expected no. of entities in the queue

$W = \dfrac{1}{\mu - \lambda}$ average waiting time in the queueing system

$W_q = \dfrac{\lambda}{(\mu - \lambda)\mu}$ average waiting time in the queue

For VMC conditions, the stochastic queueing equations yield:

  Queueing Parameters

    System utilization (%) =   86.0465

    Idle probability (dim) =   0.13953

    Expected No. of aircraft in queue (Lq) = 5.3062

    Expected No. of aircraft in system (L)   = 6.1667

    Average Waiting Time in Queue (hours) =   0.14341 hours (8.6 minutes)

    Average Waiting Time in System (includes service) =   0.16667 hours (10 minutes)

For IMC conditions, the stochastic queueing equations do not yield a steady-state solution. Therefore, a ground delays program at origin airports is needed.

For departures:

The service rates (μ) are 45 for IMC conditions and 52 for VMC conditions. Using the stochastic queueing equations.

$$\lambda_{dep} = \lambda_3 + \lambda_4$$
$$\lambda_{dep} = 43 \text{ departures per hour}$$

For IMC conditions, the stochastic queueing equations yield:

     Queueing Parameters

    System utilization (%) =   95.5556

    Idle probability (dim) =   0.044

Expected No. of vehicles in queue (Lq) = 20.54

Expected No. of vehicles in system (L)   = 21.5

 Average Waiting Time in Queue (hours) = 0.48 hours (28.7 minutes)

Average Waiting Time in System (includes service) = 0.50 hours (30.0 minutes)

For VMC conditions, the stochastic queueing equations do not yield a steady-state solution. Therefore, a ground delays program at origin airports is needed.

Queueing Parameters

System utilization (%) = 82.69

Idle probability (dim) = 0.17

Expected No. of vehicles in queue (Lq) = 3.95

Expected No. of vehicles in system (L)   = 4.78

Average Waiting Time in Queue = 0.092 hours (5.51 minutes)

Average Waiting Time in System (includes service) = 0.11 hours (6.7 minutes)

b) Create a plot of delays vs. number of operations for this airport. Vary the number of operations incrementally from a small number to near saturation capacity. Create two delay curve plots: one for arrivals and one for departures and for two conditions (VMC and IMC).
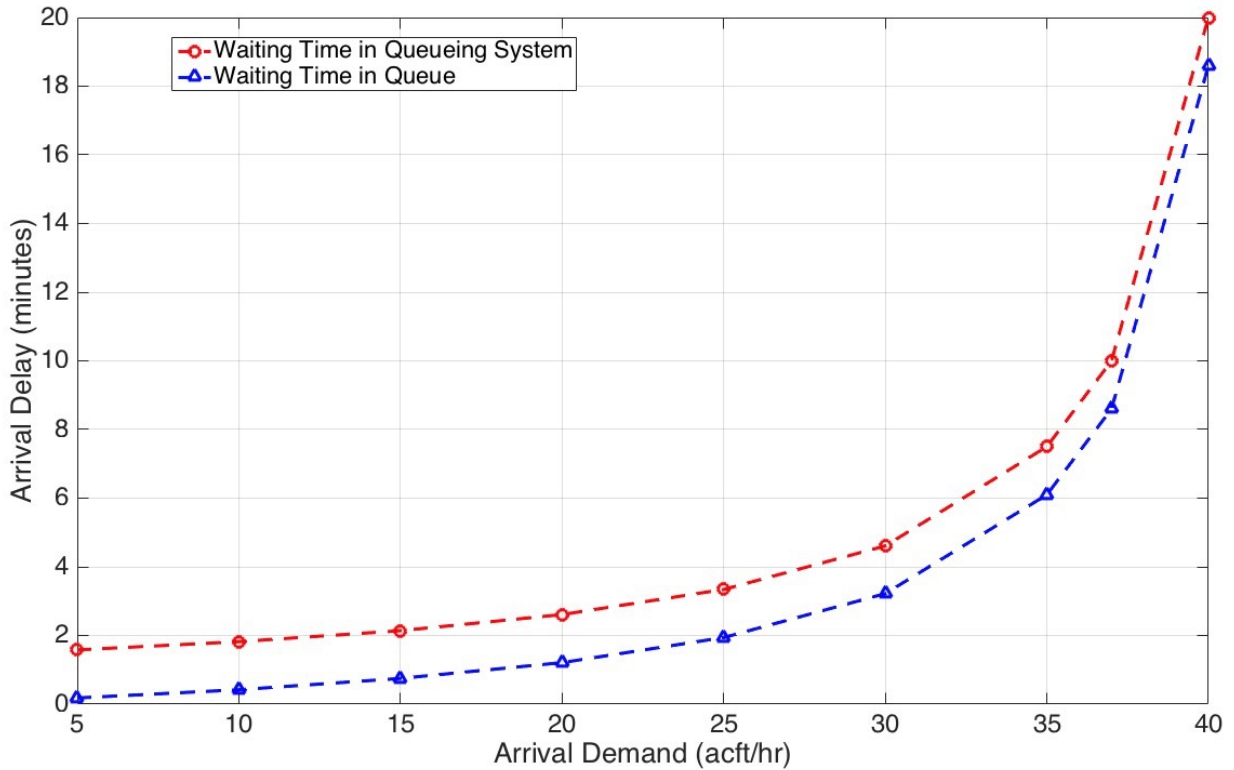

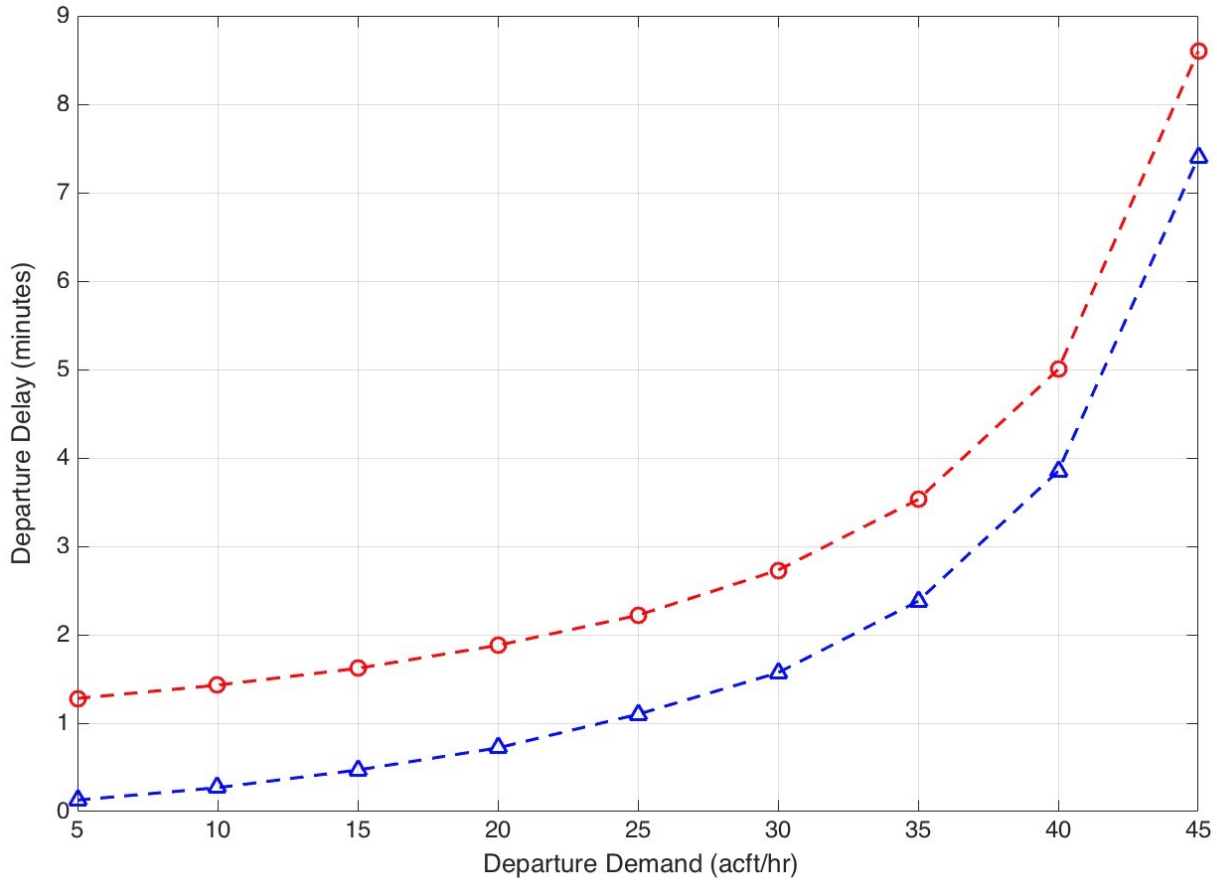Figure 1. Arrival Waiting Times as a Function of Arrival Demand. VMC Conditions.

Figure 2. Departure Waiting Times as a Function of Arrival Demand. VMC Conditions. Blue Line is the Waiting Time in Queue. Red Line is the Waiting Time in the Queueing System.

    c) If airlines want to fly continuous descent approaches from the NE and NW quadrants explain what are some of the challenges to do this. Discuss metering and timing to fly CDAs using distinct approaches and with different aircraft. Use the medium size class (http://128.173.204.63/courses/cee5614/cee5614_pub/Boeing737800Jet_class.m) aircraft to design minimum fuel consumption CDA profiles for this airport. Specify the speeds and altitudes flown from each metering fix (red dots) to the point where the aircraft intercepts the ILS glide slope (2,500 feet above sea level). Lets assume the airport is at sea level. The track from the NW quadrant fix is known to be 82 nm to the airport. The track from the NE quadrant is 78 nm. Arrivals to NE and NW fixes cross these fixes at 350 knots (TAS).

Use the unrestricted descent approach code presented in class. The solution requires some trial and error analysis.

Distance from 2,500 feet to runway (using 3 degree slope approach) = 7.9 nautical miles

Distance on NW Track for CDA = 82-7.9 = 74.1 nm

Starting altitude = 19,000 feet (5793 meters)

Starting speed = 283 knots indicated (350 knots true airspeed)

Mass = 70000 kilograms

A CDA descent for this aircraft is shown in Figure 3. Note that from 19,000 feet the aircraft descends 16,500 feet to reach 2,500 feet in 1,050 seconds. The aircraft burns 298 kilograms of fuel. The descent profile is shown in Figures 3 and 4. The speed profile in the descent was:

Vdescent = [160 180 240 250 283 283];   % knots indicated

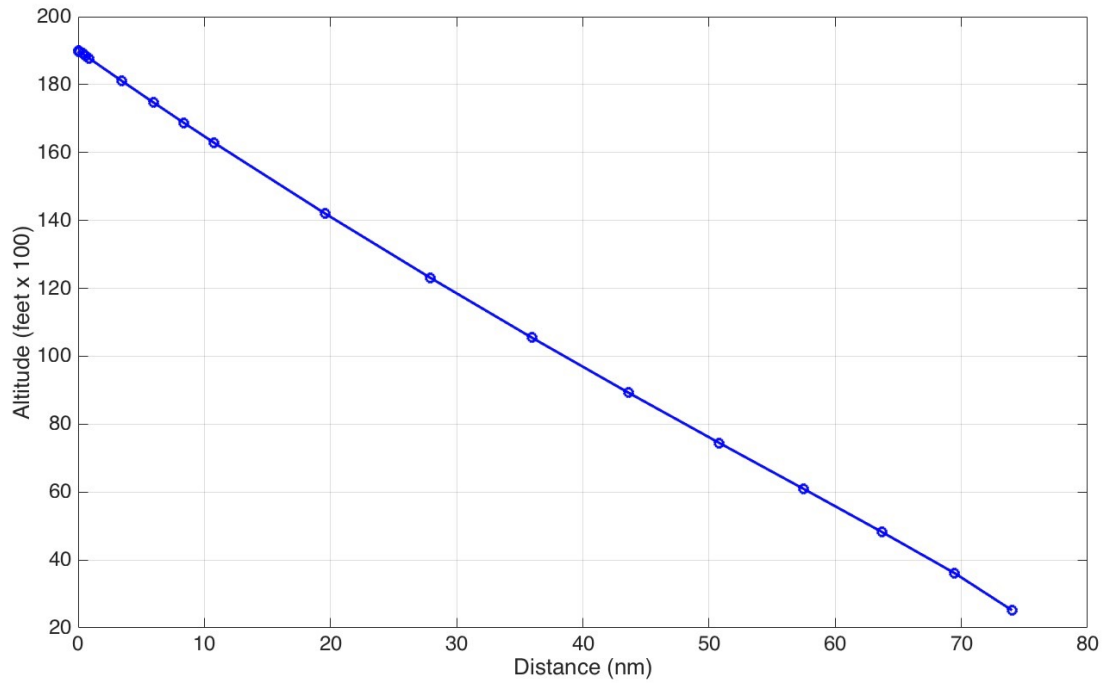altc = [762 1000 3000 4500 5793 13000];   % meters

Figure 3 CDA Trajectory from 19,000 feet (5793 meters). Mass = 70,000 kg. Fix Crossing Speed is 350 knots TAS (283 IAS). Distance to Reach 2,500 feet is 74.1 nm.
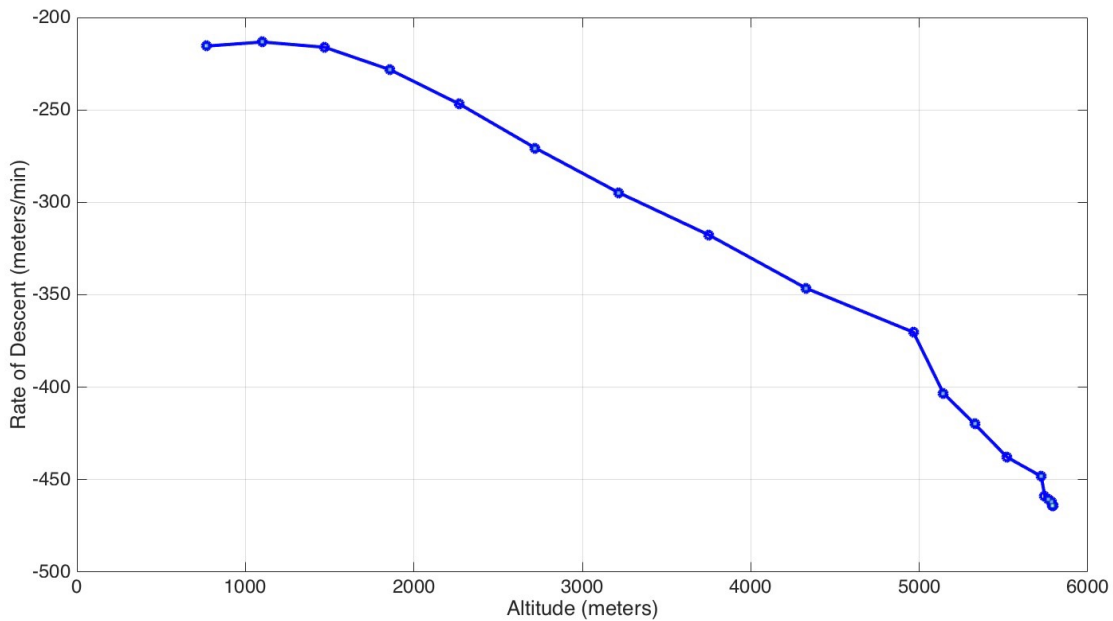

Figure 4 CDA Trajectory from 19,000 feet (5793 meters). Mass = 70,000 kg. Fix Crossing Speed is 350 knots TAS (283 IAS). Distance to Reach 2,500 feet is 74.1 nm.

Problem 2

Read the AIAA paper entitled "Three-Degree Decelerating Approaches in High-Density Arrival Streams" by de Leege et al., 2009 This paper has been published at the JOURNAL OF AIRCRAFT Vol. 46, No. 5, September–October 2009.

To be discussed in class.

Problem 3

Use the data for the large twin-engine transport jet aircraft - similar to the Boeing 777-200 (3) described in the Matlab files for CEE 5614 and answer the following questions.

The aircraft is traveling across the Atlantic from Atlanta (ATL) to London Heathrow airport (LHR). At a point 1000 nm from Gander International Airport (in Canada), the aircraft suffers a cabin pressurization system failure and the pilot initiates a diversion procedure back to Gander International Airport. At the point of failure, the aircraft has a weight of 245,000 kg. while cruising at 35,000 feet. Lacking a pressurized cabin, the pilot starts an emergency descend from its initial cruising altitude to 10,000 feet which offers a better environment for passengers to breathe normally. The aircraft emergency descent calls for a constant Indicated Airspeed (IAS) schedule of 340 knots starting at 35,000 feet and reducing the thrust of both engines to near zero.

Once leveled at 10,000 feet the aircraft cruises back to Gander at 260 knots IAS (faster speed would be unsafe in the event of bird strike at that lower altitude).

    d) Find the approximate time to reach 10,000 feet using the constant IAS speed profile.

    e) Find the time to reach Gander.

    f) Find the amount of fuel burned to get back to Gander. Explain all your assumptions.

Step # 1 Emergency descent calculations

The emergency descent used is shown in two vectors below.

Vdescent = [180 200 250 310 340 340 340 340 340 340 340 340 300 300 300];   % knots IAS

altc = [0 1000 2000 3048 4000 5000 6000 7000 8000 9000 10000 11000 12000 13000 14000]; % altitude (meters)

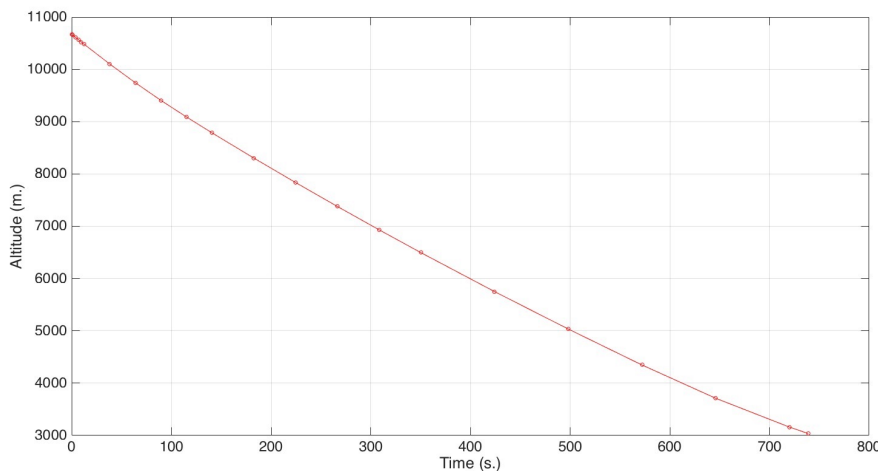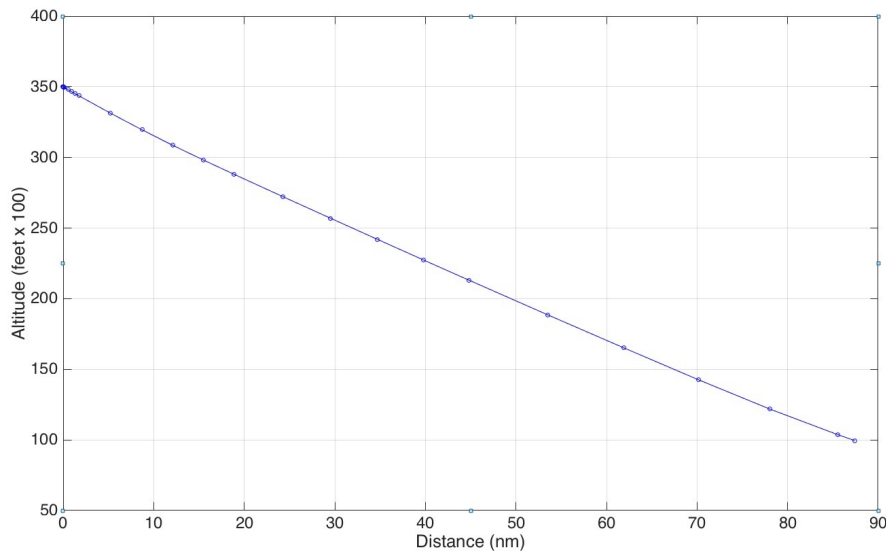Figure 5 shows the emergency descent down to 10,000 feet (3048 meters).


Figure 5 Pressurization Failure Emergency Descent from 10,671 meters (35,000 ft). Mass = 245,000 kg. IAS Speed is 340 knots TAS. Time to Reach 10,000 feet is 739 seconds (12.3 minutes).

Figure 6 Pressurization Failure Emergency Descent from 10,671 meters (35,000 ft). Initial Mass = 245,000 kg. IAS Speed is 340 knots TAS. Distance to Reach 10,000 feet is 87.2 nautical miles.

Step # 2 Cruise Calculations

During the emergency descent two engines are idle and the fuel used is estimated to be 455 kilograms. The distance remaining to cruise to Gander is 912.8 nautical miles. Assume a normal descent distance from 10,000 feet to be 65 nm (later to be revised). This leaves a total of 850.8 nm in cruise mode at 3048 meters. I used a cruise speed of 315 (IAS) knots at 10,000 feet in this solution (Mach 0.55). Recall that near airports (in the terminal area), aircraft are restricted to 250 knots IAS. The initial cruise mass is 244,545 kilograms.
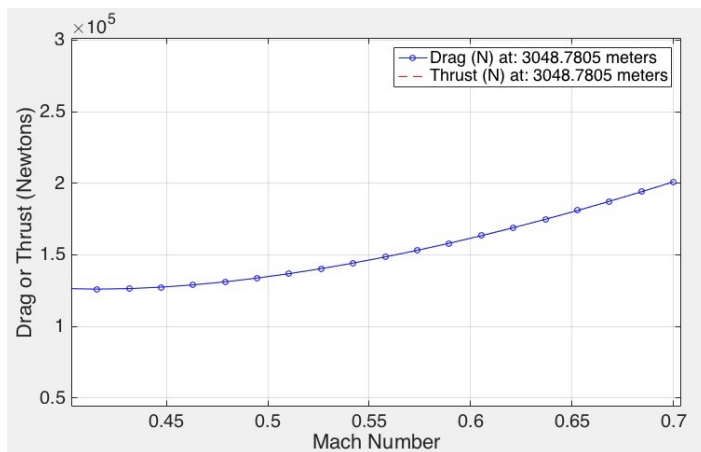
Figure 7 Drag Plot for Boeing 777 Class Aircraft as a Function of Mach Number. Initial Mass = 244,545 kg. Mach 0.55 Used for Return to Gander.

Use the cruise formulas to estimate the fuel used in cruise. Used a Matlab program using 50 segments to estimate fuel. The results are:

Cruise Fuel (kg) 20349.48

Cruise Time (minutes) 144.9

 Cruise Distance (nm) 847.8

Mass at the end of the cruise = 224,200 kilograms.

The Matlab code to do the numerical integration is:

```
% Script to estimate the amount of fuel burned in the cruise segment
% Programmer: A. Trani
% date: November 2015
% Version 1.0
clear
clc

% Given:
%
% Temperature off ISA
% Aircraft model to use
%
% Functions used by this program: drag03.m and densityOffISA.m
%
% % _____ Cruise problem _____

    % Load the aircraft model and delta temperature form ISA
    deltaTemp = 0;        % 0 = ISA
    % boeing787_class
    Boeing777_class_Q2_2015

    % Conversion factors

    kmToSM = 1.609;          % km to statute miles
    kmToNM = 1.852;          % km to nautical miles
    hcruise = 10000/3.28;     % Altitude in meters
    mach_cruise = 0.55;      % Desired cruise conditions

    % This new procedure divives the cruise phase into a numerical
    % integration procedure with noSteps as the number of steps to be used
    % in the integration process

    initialCruiseMass = 244545;          % initial mass at cruise (kg)
    currentCruiseMass(1) = initialCruiseMass;  % Define teh first cell of a new array with cruise mass
    cruiseDistance = 847.8;               % nm to reach the NAT point
    noSteps = 50;                         % Number of steps to estimate fuel burn

    cruiseDistanceInterval = cruiseDistance*   kmToNM*1000 /noSteps;     % meters

    % Start a loop to calculate the atmospheric and aerodynamic parameters

    for i=1:noSteps

        % Calculate drag at cruise speed drag03.m

        phi = 0.0;  % bank angle
        [speedOfSound,density,temp,dAlt] = densityOffISA(deltaTemp,hcruise);
        [drag(i),LD(i),cl(i),cd(i)]=drag_03(mach_cruise,density,speedOfSound,currentCruiseMass(i),macht,Cdoct,phi);
% find drag

        % Calculate fuel flow in cruise
        fuelFlowCruise(i) = drag(i) * tsfc;        % fuel flow in cruise (N/s)
```

```
        % Calculation of time in cruise
        vtas = mach_cruise * speedOfSound;              % true airspeed (m/s)
        cruiseTimeInterval = cruiseDistanceInterval / vtas ;    % time interval per cruise segement in seconds
        cruiseMassConsumed(i) = fuelFlowCruise(i) * cruiseTimeInterval / g;         % units are kilograms

        currentCruiseMass(i+1) = currentCruiseMass(i) - cruiseMassConsumed(i);      % initial mass at cruise (kg)

    end

    % Collect terms and report to Command window

    cruiseFuel = sum(cruiseMassConsumed);
    cruiseDistance = cruiseDistanceInterval*noSteps;
    cruiseTime = cruiseTimeInterval*noSteps;

    clc

    disp(' ')
    disp([' Cruise Fuel (kg)  ',num2str(cruiseFuel)])
    disp([' Cruise Time (minutes)  ',num2str((cruiseTimeInterval*noSteps)/60)])
    disp([' Cruise Distance (nm)  ',num2str((cruiseDistanceInterval*noSteps)/(1000*kmToNM))])
```
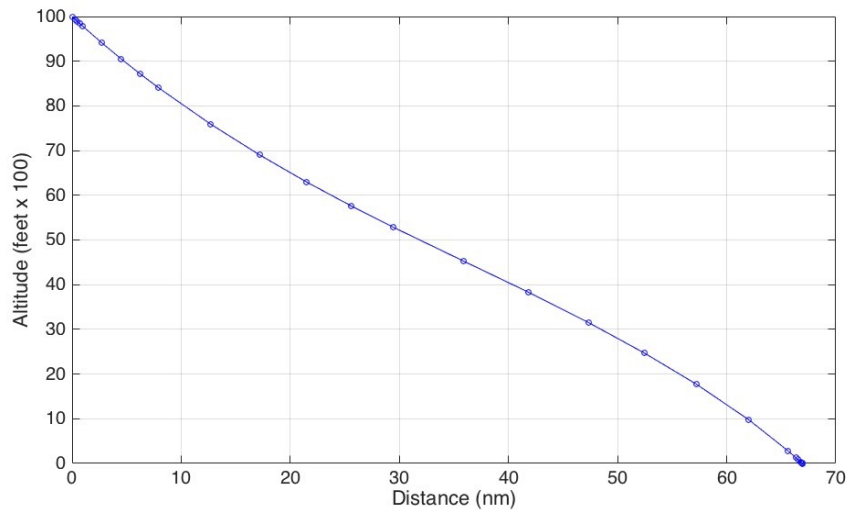
Once you obtain the results in Step # 3 you can go back and revise the solution further.

Step # 3 Final Descent Calculations

During the normal descent from 10,000 feet (3048 meters) the aircraft flies 67 nm and the fuel used is estimated to be 455 kilograms. The distance remaining to cruise to Gander is 912.8 nautical miles

Figure 8 Descent from 3,048 meters (10,000 ft). Initial Mass = 224,200 kg. Distance to Reach Airport (sea level) feet is 67 nautical miles. Fuel used = 1,385 kg. Time to Descent from 10,000 feet = 1122 seconds (18.7 minutes).

Total Travel time to Gander = 18.7 + 144.9 + 12.3 = 175.9 minutes. The detour is legal and under 180 minutes.

Total fuel used to reach Gander = 455 + 20349 + 1385 = 22,189 kilograms.