Assignment 6: Maneuvering Analysis and ATC

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Problem 1

One of the fuel savings initiatives under NextGen is to conduct Continuous Descent Approaches (CDA) to busy airports. A sample CDA approach is shown in Figure 1. The CDA approach starts x minutes later (called CDA distance offset) than the normal approach for our analysis. The aircraft initially is straight and level at 35,000 feet and flying at Mach 0.76. A typical approach to a busy airport like LAX involves one or more step down segments as shown in Figure 1. Figure 2 shows actual flight paths extracted from radar data provided by the FAA for a terminal design study. The data applies to large twin-engine transport aircraft. Note that according to Figure 2, below 10,000 feet both profiles will follow the same trajectory.

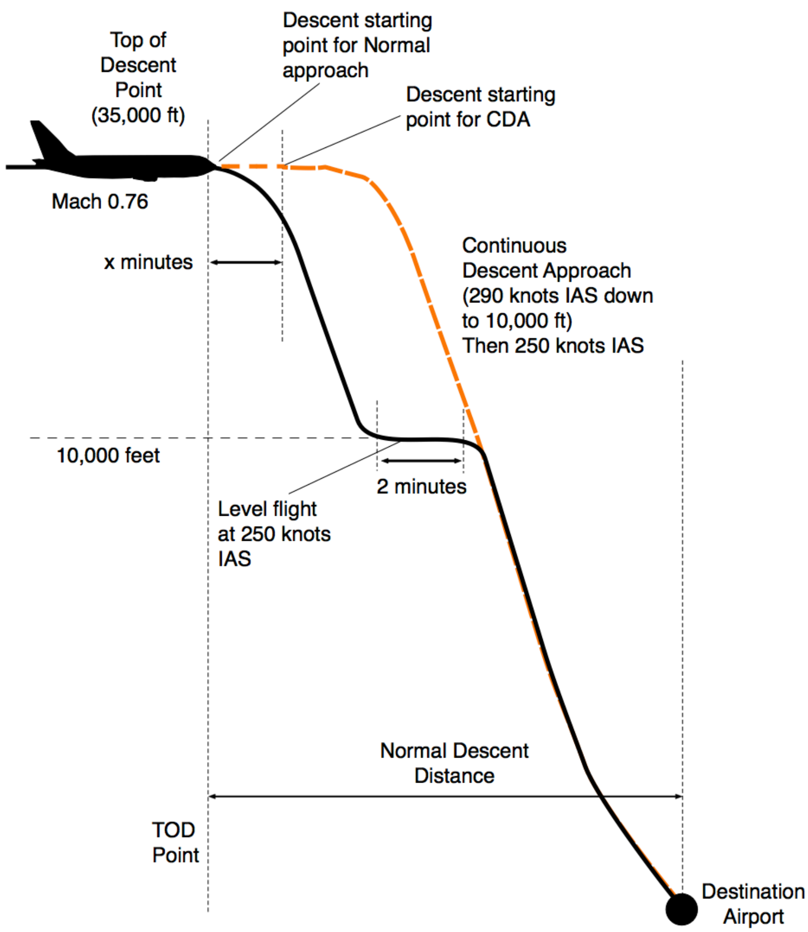

Figure 1. CDA Approach Compared to Typical Approach to LAX Airport.

a) Determine the value of x (CDA distance offset) that makes the two profiles similar below 10,000 feet.

b) Using the Boeing 737-800 class transport aircraft in our course material, estimate the fuel savings for a single flight to LAX using the CDA technique. Assume the mass at the TOD point is 58,000 kg. The speed parameters of the approach are provided in Figure 1. Assume the idle thrust is constant at 1/12 of the maximum generated at altitude. In your calculations consider the normal approach having a 2 minute step at 10,000 feet. During the step segment, the aircraft flies straight and level and thus the aircraft engines generate thrust to overcome drag.

c) Read the paper about CDA approaches by Richard Coppenbarger et al. (Journal of Aircraft, Volume 46, 2009) (http://www.aviationsystemsdivision.arc.nasa.gov/publications/2009/AIAA-39795-675.pdf) and comment on the possible fuel savings reported in the literature. Compare to your calculations.

______________________________ *Solution* _______________________

At 10,000 feet and aircraft reporting 250 knots IAS, the speed of sound is 328.4 m/s. The aircraft is traveling at Mach 0.437 speed or 143.66 m/s. In two minutes of forward travel, the aircraft travels 17,239 meters (9.301 nm). This distance is used to estimate the offset time at 35,000 feet when the aircraft travels at Mach 0.76.

At 36,000 feet the speed of sound is 295.9 m/s and the Mach number 0.76 translates into 224.9 m/s. The aircraft requires a time offset of 76.65 seconds or one minute and 16.6 seconds. Table 1 compares the fuel consumptions of the aircraft while doing the CDA procedure and the regular descent profile with a two minute holding at 10,000 feet.

Table 1. Comparison of Fuel Consumption for Two Descent Profiles.

Fuel Component	Fuel Regular Profile (kg)	Fuel CDA Profile (kg)
Level flight in cruise	0	51.03
Descent profile to 10,000 feet	200.55	200.55
Level flight holding	76.79	0
Total	**277.34**	**251.58**

a) At 35,000 feet, the Boeing 737-800 class aircraft has a drag of 34,379 N. This translates into 0.67 kg/second. The aircraft burns 51,03 kg in 76.6 seconds in level flight at 35,000 feet.

b) At 10,000 feet, the Boeing 737-800 class aircraft has a total drag of 33,041 N while traveling at 250 knots IAS. The fuel burn is 0.64 kg/second. Over 120 seconds, the total fuel used is: 76.79 kg.

c) The difference between the two profiles is close to 26 kg per descent. This over the life cycle of the vehicle could be significant given that this cycle is repeated 3-5 times a day for this type of aircraft.

___________________________ End of Solution __________________

Problem 2

On a busy day, Lindon Heathrow Airport approach control stacks many aircraft in holding patterns as shown in Figure 3. London approach control instructs pilots to fly a standard 4-minute pattern with one minute per side and one minute in the 180-degree turns. On this busy day, the controller instructs a pilot flying a narrow body aircraft (like the Boeing 737-800 class vehicle we use in class) to fly the pattern at 15,000 feet and 250 knots (IAS).

a) Estimate the fuel burn consumed in the 4-minute holding pattern for the aircraft in question.
b) Estimate the thrust setting (how much thrust is needed to keep the aircraft in steady and level flight) on both the straight and the turning segments of the holding pattern.
c) Find the radius of the turn for the aircraft flying the holding pattern.
d) What is the bank angle (in degrees) to fly the holding pattern?

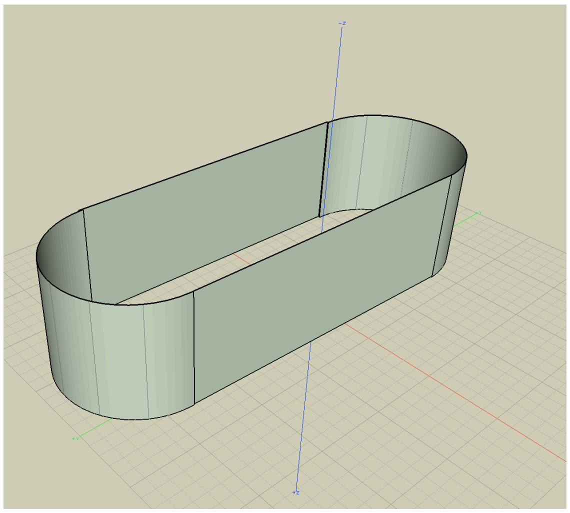

Figure 2. Sample 3D view of a Holding Pattern near an Airport.

_____________________________ *Solution* ______________________

At 15,000 feet and aircraft reporting 250 knots IAS, the speed of sound is 322.3 m/s. The aircraft is traveling at Mach 0.4721 speed or 152.13 m/s.

For the straight segments of the holding pattern: the drag flying at Mach 0.4721 is estimated for various aircraft weights as shown in Table 2.

Table 2. Fuel Consumption for Boeing 737-800 Class Aircraft. Straight Track in Segment of Holding Pattern (1 minute per side).

Aircraft Mass (kg)	Drag (N)	Fuel Burn (kg/s)	Fuel in Straight Segment (kg)
58000	3.23E+04	0.626	37.540
60000	3.32E+04	0.643	38.581
62000	3.41E+04	0.661	39.662
64000	3.51E+04	0.680	40.776

A standard turn is executed at Mach 0.4721 (152.13 m/s). Using the standard turn rate equations shown below, we can estimate the bank angle (ϕ) needed to satisfy a 3 degree per second turn (Γ).

$$n = 1/\cos(\phi)$$

$$R = \frac{v^2}{g(n^2-1)}$$

$$\Gamma = \frac{g\sqrt{n^2-1}}{v}$$

$$\frac{v\Gamma}{g} = \sqrt{n^2-1}$$

$$\left(\frac{v\Gamma}{g}\right)^2 + 1 = n^2$$

Note that the speed is 152.13 m/s, the value of turn rate is 3 deg/second and g=9.81 m/s^2. Solving for the load factor (n),

$$1.6504 = n^2$$

$$n = 1.2882$$

$$\cos(\phi) = 1/n$$

$$\phi = a\cos(1/n)$$

$$\phi = a\cos(1/1.2882)$$

$$\phi = a\cos(0.7763)$$

$$\phi = 0.6821 \text{ radians}$$

$$\phi = 39.1 \text{ degrees}$$

This bank angle exceeds the operational bank angle used by the airlines (25-30 degrees).

Using the fact that when an aircraft banks, the value of the lift coefficient needed to fly the banked aircraft increases as shown below,

$$C_l = \frac{2mg}{\rho V^2 S \cos(\phi)}$$

We recalculate the drag generated in the banking segment. The values are shown in Table 3. Observing the values in Table 3, it is clear that in the turning segments, the fuel consumption is increased substantially. For example, at 58,000 kilograms, the turning maneuver increases the fuel used by 26%. The thrust needed in the turn is just the drag produced by the turning aircraft. For example, at 60,000 kg, the thrust required to maintain a steady turn will be 42,200 Newtons (21,100 Newtons per engine).

The radius of the turn can be now calculated as follows:

$$n = 1/\cos(\phi)$$

$$R = \frac{v^2}{g(n^2 - 1)}$$

$$R = \frac{v^2}{g(1.2882^2 - 1)}$$

$$R = \frac{(152.13)^2}{6.4693}$$

$$R = 3577 \text{ meters}$$

Table 3. Fuel Consumption for Boeing 737-800 Class Aircraft. Turning Segments of Holding Pattern (1 minute per side). Bank Angle is 39.1 degrees (0.6821 radians).

Aircraft Mass (kg)	Drag (N)	Fuel Burn (kg/s)	Fuel in Turning Segment (kg)
58000	4.07E+04	0.789	47.349
60000	4.22E+04	0.818	49.080
62000	4.38E+04	0.848	50.870
64000	4.54E+04	0.879	52.719

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